gpkit Documentation

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MIT Department of Aeronautics and Astronautics

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GPkit is a Python package for defining and manipulating geometric programming (GP) models.

Our hopes are to bring the mathematics of Geometric Programming into the engineering design process in a disciplined and collaborative way, and to encourage research with and on GPs by providing an easily extensible object-oriented framework.

GPkit abstracts away the backend solver so that users can work directly with engineering equations and optimization concepts. Supported solvers are MOSEK and CVXOPT.

Join our mailing list and/or chatroom for support and examples.

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Geometric Programming 101

1.1 What is a GP?

A Geometric Program (GP) is a type of non-linear optimization problem whose objective and constraints have a particular form.

The decision variables must be strictly positive (non-zero, non-negative) quantities. This is a good fit for engineering design equations (which are often constructed to have only positive quantities), but any model with variables of unknown sign (such as forces and velocities without a predefined direction) may be difficult to express in a GP. Such models might be better expressed as *Signomials*.

More precisely, GP objectives and inequalities are formed out of *monomials* and *posynomials*. In the context of GP, a monomial is defined as:

$$f(x) = cx_1^{a_1} x_2^{a_2} ... x_n^{a_n}$$

where c is a positive constant, $x_{1..n}$ are decision variables, and $a_{1..n}$ are real exponents. For example, taking x, y and z to be positive variables, the expressions

$$7x 4xy^2z \frac{2x}{y^2z^{0.3}} \sqrt{2xy}$$

are all monomials. Building on this, a posynomial is defined as a sum of monomials:

$$g(x) = \sum_{k=1}^{K} c_k x_1^{a_1 k} x_2^{a_2 k} \dots x_n^{a_n k}$$

For example, the expressions

$$x^{2} + 2xy + 1$$
 $7xy + 0.4(yz)^{-1/3}$ $0.56 + \frac{x^{0.7}}{yz}$

are all posynomials. Alternatively, monomials can be defined as the subset of posynomials having only one term. Using f_i to represent a monomial and g_i to represent a posynomial, a GP in standard form is

written as:

minimize
$$g_0(x)$$

subject to $f_i(x) = 1, \quad i = 1,, m$
 $g_i(x) \le 1, \quad i = 1,, n$

Boyd et. al. give the following example of a GP in standard form:

$$\begin{array}{ll} \text{minimize} & x^{-1}y^{-1/2}z^{-1} + 2.3xz + 4xyz\\ \text{subject to} & (1/3)x^{-2}y^{-2} + (4/3)y^{1/2}z^{-1} \leq 1\\ & x + 2y + 3z \leq 1\\ & (1/2)xy = 1 \end{array}$$

1.2 Why are GPs special?

Geometric programs have several powerful properties:

- 1. Unlike most non-linear optimization problems, large GPs can be solved extremely quickly.
- 2. If there exists an optimal solution to a GP, it is guaranteed to be globally optimal.
- 3. Modern GP solvers require **no initial guesses** or tuning of solver parameters.

These properties arise because GPs become *convex optimization problems* via a logarithmic transformation. In addition to their mathematical benefits, recent research has shown that many practical problems can be formulated as GPs or closely approximated as GPs.

1.3 What are Signomials / Signomial Programs?

When the coefficients in a posynomial are allowed to be negative (but the variables stay strictly positive), that is called a Signomial.

A Signomial Program has signomial constraints. While they cannot be solved as quickly or to global optima, because they build on the structure of a GP they can often be solved more quickly than a generic nonlinear program. More information can be found under *Signomial Programming*.

1.4 Where can I learn more?

To learn more about GPs, refer to the following resources:

- A tutorial on geometric programming, by S. Boyd, S.J. Kim, L. Vandenberghe, and A. Hassibi.
- Convex optimization, by S. Boyd and L. Vandenberghe.
- Geometric Programming for Aircraft Design Optimization, Hoburg, Abbeel 2014

GPkit Overview

GPkit is a Python package for defining and manipulating geometric programming (GP) models, abstracting away the backend solver.

Our hopes are to bring the mathematics of Geometric Programming into the engineering design process in a disciplined and collaborative way, and to encourage research with and on GPs by providing an easily extensible object-oriented framework.

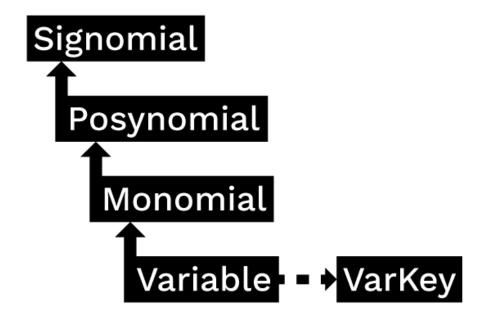
2.1 Symbolic expressions

GPkit is a limited symbolic algebra language, allowing only for the creation of geometric program compatible equations (or signomial program compatible ones, if signomial programming is enabled). As mentioned in *Geometric Programming 101*, one can view monomials as posynomials with a single term, and posynomials as signomials that have only positive coefficients. The inheritance structure of these objects in GPkit follows this mathematical basis.

2.2 Substitution

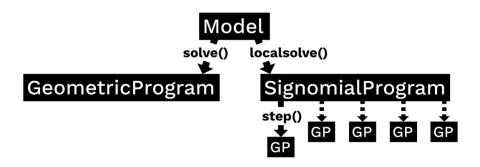
The Varkey object in the graph above is not a algebraic expression, but what GPkit uses as a variable's "name". It carries the LaTeX representation of a variable and its units, as well as any other information the user wishes to associate with a variable. The use of Varkeys as opposed to numeric indexing is an important part of the GPkit framework, because it allows a user to keep variable information local and modular.

GPkit keeps its internal representation of objects entirely symbolic until it solves. This means that any expression or Model object can replace any instance of a variable (as represented by a VarKey) with a number, new VarKey, or even an entire Monomial at any time with the .sub() method.



2.3 Model objects

In GPkit, a Model object represents a symbolic problem declaration. That problem may be either GP-compatible or SP-compatible. To avoid confusion, calling the <code>solve()</code> method on a model will either attempt to solve it for a global optimum (if it's a GP) or return an error immediately (if it's an SP). Similarly, calling <code>localsolve()</code> will either start the process of SP-solving (stepping through a sequence of GP-approximations) or return an error for GP-compatible Models. This framework is illustrated below.



2.3. Model objects 7

Installation Instructions

If you encounter bugs during installation, please email gpkit@mit.edu or raise a GitHub issue.

3.1 Installation dependencies

To install GPkit, you'll need to have the following python packages already installed on your system:

- pip
- numpy version 1.8.1 or newer
- scipy
- pint

and at least one solver, which we'll choose and install in a later step.

There are many ways to install these dependencies, but here's our suggestion:

3.1.1 **Get** pip

Mac OS X Run easy_install pip at a terminal window.

Linux

Use your package manager to install pip Ubuntu: python-pip

sudo apt-get install

Windows Install the Python 2.7 64-bit version of Anaconda.

3.1.2 Get python packages

Mac OS X

Run the following commands:

- pip install pip --upgrade
- pip install numpy
- pip install scipy
- pip install pint

Linux

Use your package manager to install numpy and scipy Ubuntu: sudo apt-get install python-numpy python-scipy

Run pip install pint (for system python installs, use sudo pip)

Windows Do nothing at this step; Anaconda already has the needed packages.

3.2 Install a GP solver

GPkit interfaces with two off the shelf solvers: cvxopt, and mosek. Cvxopt is open source; mosek requires a commercial licence or (free) academic license.

At least one solver is required.

3.2.1 Installing cvxopt

Mac OSX Run pip install cvxopt

Linux Run sudo apt-get install libblas-dev liblapack-dev libsuitesparse-dev or otherwise install those libraries

Run pip install cvxopt (for system python installs, use sudo pip)

If experiencing issues with wheel in Ubuntu 16.04, try the official installer.

Windows Run conda install -c omnia cvxopt in an Anaconda Command Prompt.

3.2.2 Installing mosek

Dependency note: GPkit uses the python package ctypesgen to interface with the MOSEK C bindings.

Licensing note: if you do not have a paid license, you will need an academic or trial license to proceed.

Mac OS X

- If which gcc does not return anything, install XCode and the Apple Command Line Tools.
- Install ctypesgen with pip install ctypesgen --pre.
- Download MOSEK, then:
 - Move the mosek folder to your home directory
 - Follow these steps for Mac.
 - Request an academic license file and put it in ~/mosek/

Linux

• Install ctypesgen with pip install ctypesgen --pre (for system python installs, use sudo pip)

Download MOSEK, then:

- Move the mosek folder to your home directory
- Follow these steps for Linux.
- Request an academic license file and put it in ~/mosek/

Windows

- Install ctypesgen by running pip install ctypesgen --pre in an Anaconda Command Prompt.
- Download MOSEK, then:
 - Follow these steps for Windows.
 - Request an academic license file and put it in
 C:\Users\((your_username)\)\\mosek\\
 - Make sure gcc is on your system path.
 - * To do this, type gcc into a command prompt.
 - * If you get executable not found, then install the 64-bit version (x86_64 installer architecture dropdown option) of mingw.
 - * Make sure the mingw bin directory is on your system path (you may have to add it manually).

3.3 Install GPkit

- Run pip install gpkit at the command line (for system python installs, use sudo pip)
- Run pip install jupyter to install jupyter notebook (recommended)
- Run jupyter nbextension enable --py widgetsnbextension for interactive control of models in jupyter (recommended)
- Run python -c "import gpkit.tests; gpkit.tests.run()" to run the tests; if any tests do not pass, please email gpkit@mit.edu or raise a GitHub issue.
- Join our mailing list and/or chatroom for support and examples.

3.4 Debugging installation

You may need to rebuild GPkit if any of the following occur:

- You install a new solver (mosek or cvxopt) after installing GPkit
- You delete the .gpkit folder from your home directory
- You see Could not load settings file. when importing GPkit, or
- Could not load MOSEK library: ImportError('\$HOME/.gpkit/expopt.so not found.')

To rebuild GPkit, first try running python -c "from gpkit.build import rebuild; rebuild()". If that do

• Run pip uninstall gpkit

3.3. Install GPkit

- Run pip install --no-cache-dir --no-deps gpkit
- Run python -c "import gpkit.tests; gpkit.tests.run()"
- If any tests fail, please email gpkit@mit.edu or raise a GitHub issue.

3.5 Bleeding-edge / developer installations

Active developers may wish to install the latest GPkit directly from the source code on Github. To do so,

- 1. Run pip uninstall gpkit to uninstall your existing GPkit.
- 2. Run git clone https://github.com/convexengineering/gpkit.git to clone the GPkit repository.
- 3. Run pip install -e gpkit to install that directory as your environment-wide GPkit.
- 4. Run cd ..; python -c "import gpkit.tests; gpkit.tests.run()" to test your installation from a non-local directory.

Getting Started

GPkit is a Python package, so we assume basic familiarity with Python: if you're new to Python we recommend you take a look at Learn Python.

Otherwise, install GPkit and import away:

```
from gpkit import Variable, VectorVariable, Model
```

4.1 Declaring Variables

Instances of the Variable class represent scalar variables. They create a VarKey to store the variable's name, units, a description, and value (if the Variable is to be held constant), as well as other metadata.

4.1.1 Free Variables

```
# Declare a variable, x
x = Variable("x")

# Declare a variable, y, with units of meters
y = Variable("y", "m")

# Declare a variable, z, with units of meters, and a description
z = Variable("z", "m", "A variable called z with units of meters")
```

4.1.2 Fixed Variables

To declare a variable with a constant value, use the Variable class, as above, but put a number before the units:

```
# Declare \rho equal to 1.225 kg/m^3.
# NOTE: in python string literals, backslashes must be doubled
rho = Variable("\\rho", 1.225, "kg/m^3", "Density of air at sea level")
```

In the example above, the key name "\rho" is for LaTeX printing (described later). The unit and description arguments are optional.

```
#Declare pi equal to 3.14
pi = Variable("\\pi", 3.14)
```

4.1.3 Vector Variables

Vector variables are represented by the VectorVariable class. The first argument is the length of the vector. All other inputs follow those of the Variable class.

```
# Declare a 3-element vector variable "x" with units of "m"
x = VectorVariable(3, "x", "m", "Cube corner coordinates")
x_min = VectorVariable(3, "x", [1, 2, 3], "m", "Cube corner minimum")
```

4.2 Creating Monomials and Posynomials

Monomial and posynomial expressions can be created using mathematical operations on variables.

```
# create a Monomial term xy^2/z
x = Variable("x")
y = Variable("y")
z = Variable("z")
m = x * y**2 / z
type(m) # gpkit.nomials.Monomial
```

```
# create a Posynomial expression x + xy^2
x = Variable("x")
y = Variable("y")
p = x + x * y**2
type(p) # gpkit.nomials.Posynomial
```

4.3 Declaring Constraints

Constraint objects represent constraints of the form Monomial >= Posynomial or Monomial == Monomial (which are the forms required for GP-compatibility).

Note that constraints must be formed using $\langle =, \rangle =$, or == operators, not \langle or \rangle .

```
# consider a block with dimensions x, y, z less than 1
# constrain surface area less than 1.0 m^2
x = Variable("x", "m")
y = Variable("y", "m")
z = Variable("z", "m")
S = Variable("S", 1.0, "m^2")
c = (2*x*y + 2*x*z + 2*y*z <= S)
type(c) # gpkit.nomials.PosynomialInequality</pre>
```

4.4 Formulating a Model

The Model class represents an optimization problem. To create one, pass an objective and list of Constraints.

By convention, the objective is the function to be *minimized*. If you wish to *maximize* a function, take its reciprocal. For example, the code below creates an objective which, when minimized, will maximize x * y * z.

```
objective = 1/(x*y*z)

constraints = [2*x*y + 2*x*z + 2*y*z \le S,

x >= 2*y]

m = Model(objective, constraints)
```

4.5 Solving the Model

When solving the model you can change the level of information that gets printed to the screen with the verbosity setting. A verbosity of 1 (the default) prints warnings and timing; a verbosity of 2 prints solver output, and a verbosity of 0 prints nothing.

```
sol = m.solve(verbosity=0)
```

4.6 Printing Results

The solution object can represent itself as a table:

```
print sol.table()
```

We can also print the optimal value and solved variables individually.

```
print "The optimal value is %s." % sol["cost"]
print "The x dimension is %s." % sol(x)
print "The y dimension is %s." % sol["variables"]["y"]
```

```
The optimal value is 15.5884619886.
The x dimension is 0.5774 meter.
The y dimension is 0.2887 meter.
```

4.7 Sensitivities and dual variables

When a GP is solved, the solver returns not just the optimal value for the problem's variables (known as the "primal solution") but also the effect that relaxing each constraint would have on the overall objective (the "dual solution").

From the dual solution GPkit computes the sensitivities for every fixed variable in the problem. This can be quite useful for seeing which constraints are most crucial, and prioritizing remodeling and assumption-checking.

4.7.1 Using variable sensitivities

Fixed variable sensitivities can be accessed from a SolutionArray's ["sensitivities"]["constants"] dict, as in this example:

```
import gpkit
x = gpkit.Variable("x")
x_min = gpkit.Variable("x_{min}", 2)
sol = gpkit.Model(x, [x_min <= x]).solve()
assert sol["sensitivities"]["constants"][x_min] == 1</pre>
```

These sensitivities are actually log derivatives $(\frac{d\log(y)}{d\log(x)})$; whereas a regular derivative is a tangent line, these are tangent monomials, so the 1 above indicates that x_{\min} has a linear relation with the objective. This is confirmed by a further example:

```
import gpkit
x = gpkit.Variable("x")
x_squared_min = gpkit.Variable("x^2_{min}", 2)
sol = gpkit.Model(x, [x_squared_min <= x**2]).solve()
assert sol["sensitivities"]["constants"][x_squared_min] == 2</pre>
```

Debugging Models

A number of errors and warnings may be raised when attempting to solve a model. A model may be primal infeasible: there is no possible solution that satisfies all constraints. A model may be dual infeasible: the optimal value of one or more variables is 0 or infinity (negative and positive infinity in logspace).

For a GP model that does not solve, solvers may be able to prove its primal or dual infeasibility, or may return an unknown status.

GPkit contains several tools for diagnosing which constraints and variables might be causing infeasibility. The first thing to do with a model m that won't solve is to run m. debug(), which will search for changes that would make the model feasible:

```
"Debug examples"

from gpkit import Variable, Model, units

x = Variable("x", "ft")
x_min = Variable("x_min", 2, "ft")
x_max = Variable("x_max", 1, "ft")
y = Variable("y", "volts")

m = Model(x/y, [x <= x_max, x >= x_min])
m.debug()

print "# Now let's try a model unsolvable with relaxed constants\n"

Model(x, [x <= units("inch"), x >= units("yard")]).debug()

print "# And one that's only unbounded\n"

# the value of x_min was used up in the previous model!
x_min = Variable("x_min", 2, "ft")
Model(x/y, [x >= x_min]).debug()
```

```
< DEBUGGING >
> Trying with bounded variables and relaxed constants:
Solves with these variables bounded:
  value near upper bound: y
sensitive to upper bound: y
and these constants relaxed:
 x_min [ft]: relaxed from 2 to 1
>> Success!
# Now let's try a model unsolvable with relaxed constants
< DEBUGGING >
> Trying with bounded variables and relaxed constants:
>> Failure.
> Trying with relaxed constraints:
Solves with these constraints relaxed:
  1: 3500% relaxed, from 3*x**-1 \le 1
                       to 3*x**-1 <= 36
>> Success!
# And one that's only unbounded
< DEBUGGING >
> Trying with bounded variables and relaxed constants:
Solves with these variables bounded:
  value near upper bound: y
sensitive to upper bound: y
>> Success!
```

Note that certain modeling errors (such as omitting or forgetting a constraint) may be difficult to diagnose from this output.

5.1 Potential errors and warnings

- RuntimeWarning: final status of solver 'mosek' was 'DUAL_INFEAS_CER', not 'optima
 - The solver found a certificate of dual infeasibility: the optimal value of one or more variables is 0 or infinity. See *Dual Infeasibility* below for debugging advice.
- RuntimeWarning: final status of solver 'mosek' was 'PRIM_INFEAS_CER', not 'optima
 - The solver found a certificate of primal infeasibility: no possible solution satisfies all constraints. See *Primal Infeasibility* below for debugging advice.
- RuntimeWarning: final status of solver 'cvxopt' was 'unknown', not 'optimal' or Run

- The solver could not solve the model or find a certificate of infeasibility. This may indicate a dual infeasible model, a primal infeasible model, or other numerical issues. Try debugging with the techniques in *Dual* and *Primal Infeasibility* below.
- RuntimeWarning: Primal solution violates constraint: 1.0000149786 is greater tha
 - this warning indicates that the solver-returned solution violates a constraint of the model, likely because the solver's tolerance for a final solution exceeds GPkit's tolerance during solution checking. This is sometimes seen in dual infeasible models, see *Dual Infeasibility* below. If you run into this, please note on this GitHub issue your solver and operating system.
- RuntimeWarning: Dual cost nan does not match primal cost 1.00122315152
 - Similarly to the above, this warning may be seen in dual infeasible models, see *Dual Infeasibility* below.

5.2 Dual Infeasibility

In some cases a model will not solve because the optimal value of one or more variables is 0 or infinity (negative or positive infinity in logspace). Such a problem is *dual infeasible* because the GP's dual problem, which determines the optimal values of the sensitivites, does not have any feasible solution. If the solver can prove that the dual is infeasible, it will return a dual infeasibility certificate. Otherwise, it may finish with a solution status of unknown.

gpkit.constraints.bounded.Bounded is a simple tool that can be used to detect unbounded variables and get dual infeasible models to solve by adding extremely large upper bounds and extremely small lower bounds to all variables in a ConstraintSet.

When a model with a Bounded ConstraintSet is solved, it checks whether any variables slid off to the bounds, notes this in the solution dictionary and prints a warning (if verbosity is greater than 0).

For example, Mosek returns DUAL_INFEAS_CER when attempting to solve the following model:

```
"Demonstrate a trivial unbounded variable"
from gpkit import Variable, Model
from gpkit.constraints.bounded import Bounded

x = Variable("x")

constraints = [x >= 1]

m = Model(1/x, constraints) # MOSEK returns DUAL_INFEAS_CER on .solve()
m = Model(1/x, Bounded(constraints))
# by default, prints bounds warning during solve
sol = m.solve(verbosity=0)
print sol.summary()
print "sol['boundedness'] is:", sol["boundedness"]
```

Upon viewing the printed output,

```
Solves with these variables bounded:
   value near upper bound: x
   sensitive to upper bound: x

(continues on next page)
```

```
Cost
---
1e-30

Free Variables
----
x: 1e+30

sol['boundedness'] is: {'value near upper bound': set([x]), 'sensitive to_
upper bound': set([x])}
```

The problem, unsurprisingly, is that the cost 1/x has no lower bound because x has no upper bound.

For details read the Bounded docstring.

5.3 Primal Infeasibility

A model is primal infeasible when there is no possible solution that satisfies all constraints. A simple example is presented below.

```
"A simple primal infeasible example"
from gpkit import Variable, Model

x = Variable("x")
y = Variable("y")

m = Model(x*y, [
    x >= 1,
    y >= 2,
    x*y >= 0.5,
    x*y <= 1.5
])

# m.solve() # raises uknown on cvxopt
    # and PRIM_INFEAS_CER on mosek</pre>
```

It is not possible for x * y to be less than 1.5 while x is greater than 1 and y is greater than 2.

A common bug in large models that use substitutions is to substitute overly constraining values in for variables that make the model primal infeasible. An example of this is given below.

```
"Another simple primal infeasible example"
from gpkit import Variable, Model

#Make the necessary Variables
x = Variable("x")
y = Variable("y", 2)

#make the constraints
constraints = [
    x >= 1,
    0.5 <= x*y,
    x*y <= 1.5</pre>
```

```
#declare the objective
objective = x*y

#construct the model
m = Model(objective, constraints)

#solve the model
#raises RuntimeWarning uknown on cvxopt and RuntimeWarning
#PRIM_INFES_CER with mosek
#m.solve()
```

Since y is now set to 2 and x can be no less than 1, it is again impossible for x * y to be less than 1.5 and the model is primal infeasible. If y was instead set to 1, the model would be feasible and the cost would be 1.

5.3.1 Relaxation

If you suspect your model is primal infeasible, you can find the nearest primal feasible version of it by relaxing constraints: either relaxing all constraints by the smallest number possible (that is, dividing the less-than side of every constraint by the same number), relaxing each constraint by its own number and minimizing the product of those numbers, or changing each constant by the smallest total percentage possible.

```
"Relaxation examples"
from gpkit import Variable, Model
x = Variable("x")
x_min = Variable("x_min", 2)
x_max = Variable("x_max", 1)
m = Model(x, [x \le x_max, x \ge x_min])
print "Original model"
print "----"
print m
print
# m.solve() # raises a RuntimeWarning!
print "With constraints relaxed equally"
print "========"
from gpkit.constraints.relax import ConstraintsRelaxedEqually
allrelaxed = ConstraintsRelaxedEqually(m)
mr1 = Model(allrelaxed.relaxvar, allrelaxed)
print mr1
print mr1.solve(verbosity=0).table() # solves with an x of 1.414
print
print "With constraints relaxed individually"
print "========="
from gpkit.constraints.relax import ConstraintsRelaxed
constraintsrelaxed = ConstraintsRelaxed(m)
mr2 = Model(constraintsrelaxed.relaxvars.prod() * m.cost**0.01,
           # add a bit of the original cost in
           constraintsrelaxed)
print mr2
```

```
Original model
 # minimize
  # subject to
       x <= x_max
       x >= x_min
With constraints relaxed equally
_____
  # minimize
      C_Relax
  # subject to
       C_Relax >= x*x_max**-1
       C_Relax >= x**-1*x_min
       C_Relax >= 1
Cost
1.414
Free Variables
x : 1.414
 | Relax
C: 1.414
Constants
x_max : 1
x_min : 2
Sensitivities
x_{min} : +0.5
x_max : -0.5
With constraints relaxed individually
```

```
_____
 # minimize
       C_{Relax.1_(0,)} *C_{Relax.1_(1,)} *x**0.01
 # subject to
       C_{Relax.1} \ge [gpkit.Monomial(x*x_max**-1), gpkit.Monomial(x**-1*x_
→min)]
       C_Relax.1 >= 1
Cost
2
Free Variables
x : 1
| Relax.1
C : [ 1
Constants
x_max : 1
x_min : 2
Sensitivities
_____
x min : +1
x_max : -0.99
With constants relaxed individually
_____
 # minimize
      x**0.01*x_max_Relax.2*x_min_Relax.2
 # subject to
      x \le x_max
       x >= x_min
       x_{min}_{Relax.2} >= 1
       x_min >= x_min_Relax.2**-1*x_min_{before}_Relax.2
       x_min <= x_min_Relax.2*x_min_{before}_Relax.2</pre>
       x_max_Relax.2 >= 1
       x_max \ge x_max_Relax.2**-1*x_max_{before}_Relax.2
       x_max <= x_max_Relax.2*x_max_{before}_Relax.2</pre>
Cost
Free Variables
_____
  x : 1
x max : 1
x_min : 1
    | Relax.2
```

```
x_max : 1
x_min : 2

Constants
-----
x_max_{before} : 1
x_min_{before} : 2

Sensitivities
------
x_min : +1
x_max : -0.99
```

Visualization and Interaction

6.1 Sankey Diagrams

6.1.1 Requirements

- jupyter notebook
- · ipysankeywidget

6.1.2 Example

Code in this section uses the CE solar model

```
from solar import Mission
M = Mission(latitude=[20], sp=False)
M.cost = M.solar.Wtotal
del M.substitutions[M.solar.wing.planform.tau]
sol = M.solve("mosek")

from gpkit.interactive.sankey import Sankey
Sankey(M).diagram(M.solar.Wtotal)
```

```
(objective) adds +1 to the sensitivity of Wtotal_Mission/Aircraft
(objective) is Wtotal_Mission/Aircraft [lbf]

adds +2.71 to the overall sensitivity of Wtotal_Mission/Aircraft
is Wtotal_Mission/Aircraft <= 0.5*CL_Mission/FlightSegment/AircraftPerf/
→WingAero*S_Mission/Aircraft/Wing/Planform.2*V_Mission/FlightSegment/
→FlightState**2*rho_Mission/FlightSegment/FlightState</pre>
```

```
adds -3.71 to the overall sensitivity of Wtotal_Mission/Aircraft is Wtotal_Mission/Aircraft >= W_Mission/Aircraft/Battery + W_Mission/

→Aircraft/Empennage + W_Mission/Aircraft/Motor + W_Mission/Aircraft/

→SolarCells + W_Mission/Aircraft/Wing + Wavn_Mission/Aircraft + Wpay_

→Mission/Aircraft
```

6.1.3 Explanation

Sankey diagrams can be used to visualize sensitivity structure in a model. A blue flow from a constraint to its parent indicates that the sensitivity of the chosen variable (or of making the constraint easier, if no variable is given) is negative; that is, the objective of the overall model would improve if that variable's value were increased *in that constraint alone*. Red indicates a positive sensitivity: the objective and the the constraint 'want' that variable's value decreased. Gray flows indicate a sensitivity whose absolute value is below 1e-7, i.e. a constraint that is inactive for that variable. Where equal red and blue flows meet, they cancel each other out to gray.

6.1.4 Usage

Variables

In a Sankey diagram of a variable, the variable is on the left with its final sensitivity; to the right of it are all constraints that variable is in.

Free

Free variables have an overall sensitivity of 0, so this visualization shows how the various pressures on that variable in all its constraints cancel each other out; this can get quite complex, as in this diagram of the pressures on wingspan:

```
Sankey(M).sorted_by("constraints", 1)
```

Fixed

Fixed variables can have a nonzero overall sensitivity. Sankey diagrams can how that sensitivity comes together:

```
Sankey(M).diagram(M.mission[0].loading[1].vgust)
```

Equivalent Variables

If any variables are equal to the diagram's variable (modulo some constant factor; e.g. $2 \times x = y$ counts for this, as does $2 \times x < y$ if the constraint is sensitive), they are found and plotted at the same time, and all shown on the left. The constraints responsible for this are shown next to their labels.

```
Sankey(M).diagram(M["CLstall"])
```

Models

When created without a variable, the diagram shows the sensitivity of every named model to becoming locally easier. Because derivatives are additive, these sensitivities are too: a model's sensitivity is equal to the sum of its constraints' sensitivities. Gray lines in this diagram indicate models without any tight constraints.

```
Sankey(M).diagram(left=60, right=90, width=1050)
```

6.1.5 Syntax

Code	Result	
s = Sankey(M)	Creates Sankey object of a given model	
s.diagram(vars)	Creates the diagram in a way Jupyter knows how to present	
d = s.diagram()	Don't do this! Captures output, preventing Jupyter from seeing it.	
s.diagram(width=.	Sets width in pixels. Same for height.	
)		
s.diagram(left=	Sets top margin in pixels. Same for right, top. bottom. Use if the left-	
.)	hand text is being cut off.	
S.	Shows the variable / top constraint on the right instead of the left.	
diagram(flowright=T	diagram(flowright=True)	
S.	Creates diagram of the variable with the largest single constraint sensi-	
sorted_by ("maxflow", tivity. (change the 0 index to go down the list)		
0)		
s.	Creates diagram of the variable that's in the most constraints. (change	
sorted_by ("constrainthe"), index to go down the list)		
0)		

6.2 Plotting a 1D Sweep

Methods exist to facilitate creating, solving, and plotting the results of a single-variable sweep (see *Sweeps* for details). Example usage is as follows:

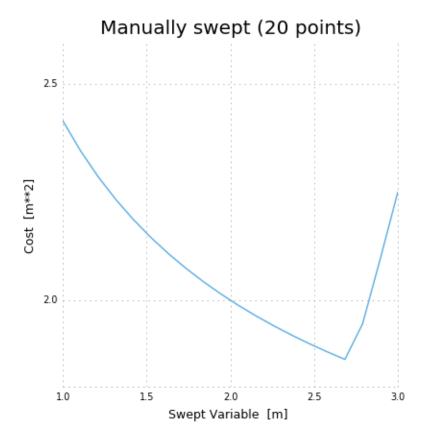
```
"Demonstrates manual and auto sweeping and plotting"
import matplotlib as mp1
mpl.use('Agg')
# comment out the lines above to show figures in a window
import numpy as np
from gpkit import Model, Variable, units

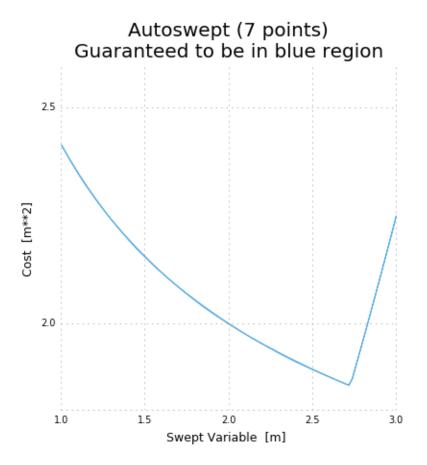
x = Variable("x", "m", "Swept Variable")
y = Variable("y", "m^2", "Cost")
m = Model(y, [y >= (x/2)**-0.5 * units.m**2.5 + 1*units.m**2, y >= (x/2)**2])
```

```
# arguments are: model, swept: values, posnomial for y-axis
sol = m.sweep({x: np.linspace(1, 3, 20)}, verbosity=0)
f, ax = sol.plot(y)
ax.set_title("Manually swept (20 points)")
f.show()
f.savefig("plot_sweep1d.png")

# arguments are: model, swept: (min, max, optional logtol), posnomial for y-
axis
sol = m.autosweep({x: (1, 3)}, tol=0.001, verbosity=0)
f, ax = sol.plot(y)
ax.set_title("Autoswept (7 points)\nGuaranteed to be in blue region")
f.show()
f.savefig("plot_autosweep1d.png")
```

Which results in:





Building Complex Models

7.1 Inheriting from Model

GPkit encourages an object-oriented modeling approach, where the modeler creates objects that inherit from Model to break large systems down into subsystems and analysis domains. The benefits of this approach include modularity, reusability, and the ability to more closely follow mental models of system hierarchy. For example: two different models for a simple beam, designed by different modelers, should be able to be used interchangeably inside another subsystem (such as an aircraft wing) without either modeler having to write specifically with that use in mind.

When you create a class that inherits from Model, write a .setup() method to create the model's variables and return its constraints. GPkit.Model.__init__ will call that method and automatically add your model's name and unique ID to any created variables.

Variables created in a setup method are added to the model even if they are not present in any constraints. This allows for simplistic 'template' models, which assume constant values for parameters and can grow incrementally in complexity as those variables are freed.

At the end of this page a detailed example shows this technique in practice.

7.2 Accessing Variables in Models

GPkit provides several ways to access a Variable in a Model (or ConstraintSet):

- using Model.variables_byname(key). This returns all Variables in the Model, as well as in any submodels, that match the key.
- using Model.topvar(key). This returns the top-level Variable that matches the key. The Variable must appear at the top level, not in a submodel.
- using Model.__getitem__. Model [key] returns the only variable matching the key, even if the match occurs in a submodel. If multiple variables match the key, an error is raised.

These methods are illustrated in the following example.

```
"Demo of accessing variables in models"
from gpkit import Model, Variable
class Battery(Model):
   """A simple battery
   Upper Unbounded
   Lower Unbounded
    .....
   def setup(self):
       h = Variable("h", 200, "Wh/kg", "specific energy")
       E = self.E = Variable("E", "MJ", "stored energy")
       m = self.m = Variable("m", "lb", "battery mass")
       return [E <= m*h]
class Motor(Model):
   """Electric motor
   Upper Unbounded
   Lower Unbounded
   Pmax
    .....
   def setup(self):
       m = self.m = Variable("m", "lb", "motor mass")
       f = Variable("f", 20, "lb/hp", "mass per unit power")
       Pmax = self.Pmax = Variable("P_{max}", "hp", "max output power")
        return [m >= f*Pmax]
class PowerSystem(Model):
    """A battery powering a motor
   Upper Unbounded
   Lower Unbounded
   E, Pmax
   def setup(self):
       battery, motor = Battery(), Motor()
       components = [battery, motor]
```

```
Getting the only var 'E': E_PowerSystem/Battery [MJ]

The top-level var 'm': m_PowerSystem [lb]

All the variables 'm': [gpkit.Variable(m_PowerSystem [lb]), gpkit.

Variable(m_PowerSystem/Battery [lb]), gpkit.Variable(m_PowerSystem/Motor_

[lb])]
```

7.3 Vectorization

gpkit. Vectorize creates an environment in which Variables are created with an additional dimension:

This allows models written with scalar constraints to be created with vector constraints:

(continues on next page)

7.3. Vectorization 33

```
m.cost = m["x"]
print m.solve(verbosity=0).summary()

print "______\n"
print "VECTORIZED"
with Vectorize(3):
    m = Test()
m.cost = m["x"].prod()
m.append(m["x"][1] >= 2)
print m.solve(verbosity=0).summary()
```

7.4 Multipoint analysis modeling

In many engineering models, there is a physical object that is operated in multiple conditions. Some variables correspond to the design of the object (size, weight, construction) while others are vectorized over the different conditions (speed, temperature, altitude). By combining named models and vectorization we can create intuitive representations of these systems while maintaining modularity and interoperability.

In the example below, the models Aircraft and Wing have a .dynamic() method which creates instances of AircraftPerformance and WingAero, respectively. The Aircraft and Wing models create variables, such as size and weight without fuel, that represent a physical object. The dynamic models create properties that change based on the flight conditions, such as drag and fuel weight.

This means that when an aircraft is being optimized for a mission, you can create the aircraft (AC in this example) and then pass it to a Mission model which can create vectorized aircraft performance models for each flight segment and/or flight condition.

```
"""Modular aircraft concept"""
import numpy as np
from gpkit import Model, Vectorize, parse_variables
```

```
class AircraftP(Model):
    """Aircraft flight physics: weight <= lift, fuel burn
    Variables
    Wfuel [lbf] fuel weight
    Wburn [lbf] segment fuel burn
    Upper Unbounded
    Wburn, aircraft.wing.c, aircraft.wing.A
    Lower Unbounded
    Wfuel, aircraft.W
    def setup(self, aircraft, state):
       self.aircraft = aircraft
       self.state = state
       exec parse_variables(AircraftP.__doc__)
       self.wing_aero = aircraft.wing.dynamic(aircraft.wing, state)
       self.perf_models = [self.wing_aero]
       W = aircraft.W
       S = aircraft.wing.S
       V = state.V
       rho = state.rho
       D = self.wing_aero.D
       CL = self.wing_aero.CL
        return [W + Wfuel <= 0.5*rho*CL*S*V**2,</pre>
               Wburn >= 0.1*D], self.perf_models
class Aircraft (Model):
    """The vehicle model
   Variables
    W [lbf] weight
   Upper Unbounded
   Lower Unbounded
    _____
   wing.c, wing.S
   def setup(self):
       exec parse_variables(Aircraft.__doc__)
        self.fuse = Fuselage()
```

```
self.wing = Wing()
        self.components = [self.fuse, self.wing]
        return self.components, W >= sum(c.W for c in self.components)
    dynamic = AircraftP
class FlightState(Model):
    """Context for evaluating flight physics
   Variables
               [knots] true airspeed
    V 40
   mu 1.628e-5 [N*s/m^2] dynamic viscosity
    rho 0.74 [kg/m<sup>3</sup>] air density
    n n n
    def setup(self):
       exec parse_variables(FlightState.__doc__)
class FlightSegment (Model):
    """Combines a context (flight state) and a component (the aircraft)
    Upper Unbounded
    Wburn, aircraft.wing.c, aircraft.wing.A
   Lower Unbounded
    Wfuel, aircraft.W
    def setup(self, aircraft):
       self.aircraft = aircraft
       self.flightstate = FlightState()
       self.aircraftp = aircraft.dynamic(aircraft, self.flightstate)
       self.Wburn = self.aircraftp.Wburn
        self.Wfuel = self.aircraftp.Wfuel
        return self.flightstate, self.aircraftp
class Mission(Model):
    """A sequence of flight segments
    Upper Unbounded
    aircraft.wing.c, aircraft.wing.A
   Lower Unbounded
    aircraft.W
```

```
def setup(self, aircraft):
        self.aircraft = aircraft
        with Vectorize(4): # four flight segments
            self.fs = FlightSegment(aircraft)
        Wburn = self.fs.aircraftp.Wburn
        Wfuel = self.fs.aircraftp.Wfuel
        self.takeoff_fuel = Wfuel[0]
        return self.fs, [Wfuel[:-1] >= Wfuel[1:] + Wburn[:-1],
                         Wfuel[-1] >= Wburn[-1]]
class WingAero (Model) :
    """Wing aerodynamics
    Variables
        [-]
                 drag coefficient
                  lift coefficient
    e 0.9 [-]
                 Oswald efficiency
                  Reynold's number
   Re
       [-]
           [lbf] drag force
   Upper Unbounded
   D, wing.c, wing.A
   Lower Unbounded
    CL, wing.S
   def setup(self, wing, state):
       self.wing = wing
       exec parse_variables(WingAero.__doc__)
       c = wing.c
       A = wing.A
       S = winq.S
       rho = state.rho
       V = state.V
       mu = state.mu
        return [
           CD >= 0.074/Re**0.2 + CL**2/np.pi/A/e,
            Re == rho * V * c/mu,
            D >= 0.5 * rho * V * * 2 * CD * S1
class Wing(Model):
    """Aircraft wing model
    Variables
            [lbf]
                         weight
             [ft^2]
                         surface area
```

```
rho 1 [lbf/ft^2] areal density
        27 [-]
                       aspect ratio
    A
           [ft]
                       mean chord
    C
   Upper Unbounded
   Lower Unbounded
   def setup(self):
       exec parse_variables(Wing.__doc__)
       return [W >= S*rho, c == (S/A)**0.5]
   dynamic = WingAero
class Fuselage (Model) :
    """The thing that carries the fuel, engine, and payload
   A full model is left as an exercise for the reader.
   Variables
    _____
   W 100 [lbf] weight
   def setup(self):
       exec parse_variables(Fuselage.__doc__)
AC = Aircraft()
MISSION = Mission(AC)
M = Model(MISSION.takeoff_fuel, [MISSION, AC])
sol = M.solve(verbosity=0)
vars_of_interest = set(AC.varkeys)
vars_of_interest.update(MISSION.fs.aircraftp.unique_varkeys)
vars_of_interest.add("D")
print sol.summary(vars_of_interest)
```

Note that the output table can be filtered with a list of variables to show.

```
Cost
---
1.091 [lbf]

Free Variables
----
| Aircraft
W: 144.1 [lbf] weight
| Aircraft/Wing
S: 44.14 [ft**2] surface area
W: 44.14 [lbf] weight
(continues on next page)
```

```
c: 1.279
                                           [ft]
                                                 mean chord
    | Mission/FlightSegment/AircraftP
| Mission/FlightSegment/AircraftP/WingAero
   D: [2.74 2.73 2.72 2.72 ] [lbf] drag force
Sensitivities
  | Aircraft/Fuselage
W : +0.97
                  weight
 | Aircraft/Wing
A: -0.67
                  aspect ratio
rho : +0.43
                  areal density
Next Largest Sensitivities
  | Mission/FlightSegment/AircraftP/WingAero
 e: \begin{bmatrix} -0.18 & -0.18 & -0.18 & -0.18 \end{bmatrix} Oswald efficiency
  | Mission/FlightSegment/FlightState
 V: \begin{bmatrix} -0.22 & -0.21 & -0.21 & -0.21 \end{bmatrix} true airspeed
                      -0.11
rho : [-0.12]
              -0.11
                              -0.11 ] air density
```

CHAPTER 8

Advanced Commands

8.1 Derived Variables

8.1.1 Evaluated Fixed Variables

Some fixed variables may be derived from the values of other fixed variables. For example, air density, viscosity, and temperature are functions of altitude. These can be represented by a substitution or value that is a one-argument function accepting model.substitutions (for details, see *Substitutions* below).

```
# code from t_GPSubs.test_calcconst in tests/t_sub.py
x = Variable("x", "hours")
t_day = Variable("t_{day}", 12, "hours")
t_night = Variable("t_{night}", lambda c: 24 - c[t_day], "hours")
# note that t_night has a function as its value
m = Model(x, [x >= t_day, x >= t_night])
sol = m.solve(verbosity=0)
self.assertAlmostEqual(sol(t_night)/gpkit.ureg.hours, 12)
m.substitutions.update({t_day: ("sweep", [8, 12, 16])})
sol = m.solve(verbosity=0)
self.assertEqual(len(sol["cost"]), 3)
npt.assert_allclose(sol(t_day) + sol(t_night), 24)
```

8.1.2 Evaluated Free Variables

Some free variables may be evaluated from the values of other (non-evaluated) free variables after the optimization is performed. For example, if the efficiency ν of a motor is not a GP-compatible variable, but $(1-\nu)$ is a valid GP variable, then ν can be calculated after solving. These evaluated free variables can be represented by a Variable with evalfn metadata. Note that this variable should not be used in constructing your model!

```
# code from t_constraints.test_evalfn in tests/t_sub.py
x = Variable("x")
x2 = Variable("x^2", evalfn=lambda v: v[x]**2)
m = Model(x, [x >= 2])
m.unique_varkeys = set([x2.key])
sol = m.solve(verbosity=0)
self.assertAlmostEqual(sol(x2), sol(x)**2)
```

For evaluated variables that can be used during a solution, see externalfn under *Sequential Geometric Programs*.

8.2 Sweeps

Sweeps are useful for analyzing tradeoff surfaces. A sweep "value" is an Iterable of numbers, e.g. [1, 2, 3]. The simplest way to sweep a model is to call model.sweep({sweepvar: sweepvalues}), which will return a solution array but not change the model's substitutions dictionary. If multiple sweepvars are given, the method will run them all as independent one-dimensional sweeps and return a list of one solution per sweep. The method model.autosweep({sweepvar: (start, end)}, tol=0.01) behaves very similarly, except that only the bounds of the sweep need be specified and the region in betwen will be swept to a maximum possible error of tol in the log of the cost. For details see *ID Autosweeps* below.

8.2.1 Sweep Substitutions

Alternatively, or to sweep a higher-dimensional grid, Variables can swept with a substitution value takes the form ('sweep', Iterable), such as ('sweep', np.linspace(1e6, 1e7, 100)). During variable declaration, giving an Iterable value for a Variable is assumed to be giving it a sweep value: for example, x = Variable("x", [1, 2, 3]) will sweep x over three values.

Vector variables may also be substituted for: $\{y: ("sweep", [[1, 2], [1, 2], [1, 2]) \}$ will sweep $y \forall y_i \in \{1, 2\}$. These sweeps cannot be specified during Variable creation.

A Model with sweep substitutions will solve for all possible combinations: e.g., if there's a variable x with value ('sweep', [1, 3]) and a variable y with value ('sweep', [14, 17]) then the gp will be solved four times, for $(x,y) \in \{(1,14), (1,17), (3,14), (3,17)\}$. The returned solutions will be a one-dimensional array (or 2-D for vector variables), accessed in the usual way.

8.2.2 1D Autosweeps

If you're only sweeping over a single variable, autosweeping lets you specify a tolerance for cost error instead of a number of exact positions to solve at. GPkit will then search the sweep segment for a locally optimal number of sweeps that can guarantee a max absolute error on the log of the cost.

Accessing variable and cost values from an autosweep is slightly different, as can be seen in this example:

```
"Show autosweep_1d functionality"
import numpy as np
import gpkit
from gpkit import units, Variable, Model
from gpkit.tools.autosweep import autosweep_1d
from gpkit.small_scripts import mag
```

```
A = Variable("A", "m**2")
l = Variable("1", "m")
m1 = Model(A**2, [A >= 1**2 + units.m**2])
t.ol1 = 1e-3
bst1 = autosweep_1d(m1, tol1, l, [1, 10], verbosity=0)
print "Solved after %2i passes, cost logtol +/-%.3q" % (bst1.nsols, bst1.tol)
# autosweep solution accessing
l_vals = np.linspace(1, 10, 10)
sol1 = bst1.sample_at(l_vals)
print "values of 1:", 1_vals
print "values of A:", sol1("A")
cost_estimate = sol1["cost"]
cost_lb, cost_ub = soll.cost_lb(), soll.cost_ub()
print "cost lower bound:", cost_lb
print "cost estimate: ", cost_estimate
print "cost upper bound:", cost_ub
# you can evaluate arbitrary posynomials
np.testing.assert_allclose(mag(2*sol1(A)), mag(sol1(2*A)))
assert (sol1["cost"] == sol1(A**2)).all()
# the cost estimate is the logspace mean of its upper and lower bounds
np.testing.assert_allclose((np.log(mag(cost_lb)) + np.log(mag(cost_ub)))/2,
                           np.log(mag(cost_estimate)))
# this problem is two intersecting lines in logspace
m2 = Model(A**2, [A >= (1/3)**2, A >= (1/3)**0.5 * units.m**1.5])
tol2 = {"mosek": 1e-12, "cvxopt": 1e-7,
        "mosek_cli": 1e-6}[gpkit.settings["default_solver"]]
bst2 = autosweep_1d(m2, tol2, l, [1, 10], verbosity=0)
print "Solved after %2i passes, cost logtol +/-%.3g" % (bst2.nsols, bst2.tol)
print "Table of solutions used in the autosweep:"
print bst2.solarray.table()
```

If you need access to the raw solutions arrays, the smallest simplex tree containing any given point can be gotten with min_bst = bst.min_bst(val), the extents of that tree with bst.bounds and solutions of that tree with bst.sols. More information is in help(bst).

8.3 Tight ConstraintSets

Tight ConstraintSets will warn if any inequalities they contain are not tight (that is, the right side does not equal the left side) after solving. This is useful when you know that a constraint _should_ be tight for a given model, but reprenting it as an equality would be non-convex.

8.4 Substitutions

Substitutions are a general-purpose way to change every instance of one variable into either a number or another variable.

8.4.1 Substituting into Posynomials, Nomial Arrays, and GPs

The examples below all use Posynomials and NomialArrays, but the syntax is identical for GPs (except when it comes to sweep variables).

```
# adapted from t_sub.py / t_NomialSubs / test_Basic
from gpkit import Variable
x = Variable("x")
p = x**2
assert p.sub(x, 3) == 9
assert p.sub(x.varkeys["x"], 3) == 9
assert p.sub("x", 3) == 9
```

Here the variable x is being replaced with 3 in three ways: first by substituting for x directly, then by substituting for the VarKey("x"), then by substituting the string "x". In all cases the substitution is understood as being with the VarKey: when a variable is passed in the VarKey is pulled out of it, and when a string is passed in it is used as an argument to the Posynomial's Varkeys dictionary.

8.4.2 Substituting multiple values

```
# adapted from t_sub.py / t_NomialSubs / test_Vector
from gpkit import Variable, VectorVariable
x = Variable("x")
y = Variable("y")
z = VectorVariable(2, "z")
p = x*y*z
assert all(p.sub({x: 1, "y": 2}) == 2*z)
assert all(p.sub({x: 1, y: 2, "z": [1, 2]}) == z.sub(z, [2, 4]))
```

To substitute in multiple variables, pass them in as a dictionary where the keys are what will be replaced and values are what it will be replaced with. Note that you can also substitute for VectorVariables by their name or by their NomialArray.

8.4.3 Substituting with nonnumeric values

You can also substitute in sweep variables (see Sweeps), strings, and monomials:

```
# adapted from t_sub.py / t_NomialSubs
from gpkit import Variable
from gpkit.small_scripts import mag

x = Variable("x", "m")
xvk = x.varkeys.values()[0]
descr_before = x.exp.keys()[0].descr
y = Variable("y", "km")
yvk = y.varkeys.values()[0]
for x_ in ["x", xvk, x]:
```

```
for y_ in ["y", yvk, y]:
    if not isinstance(y_, str) and type(xvk.units) != str:
        expected = 0.001
    else:
        expected = 1.0
        assert abs(expected - mag(x.sub(x_, y_).c)) < 1e-6
if type(xvk.units) != str:
    # this means units are enabled
    z = Variable("z", "s")
    # y.sub(y, z) will raise ValueError due to unit mismatch</pre>
```

Note that units are preserved, and that the value can be either a string (in which case it just renames the variable), a varkey (in which case it changes its description, including the name) or a Monomial (in which case it substitutes for the variable with a new monomial).

8.4.4 Updating ConstraintSet substitutions

ConstraintSets have a .substitutions KeyDict attribute which will be substituted before solving. This KeyDict accepts variable names, VarKeys, and Variable objects as keys, and can be updated (or deleted from) like a regular Python dictionary to change the substitutions that will be used at solve-time. If a ConstraintSet itself contains ConstraintSets, it and all its elements share pointers to the same substitutions dictionary object, so that updating any one of them will update all of them.

8.4.5 Substituting with replacement

Any of the substitutions above can be run with p.subinplace(*args) to substitute directly into the object in question.

8.4.6 Fixed Variables

When a Model is created, any fixed Variables are used to form a dictionary: {var: var.descr["value"] for var in self.varlocs if "value" in var.descr}. This dictionary in then substituted into the Model's cost and constraints before the substitutions argument is (and hence values are supplanted by any later substitutions).

solution.subinto(p) will substitute the solution(s) for variables into the posynomial p, returning a NomialArray. For a non-swept solution, this is equivalent to p. sub(solution["variables"]).

You can also substitute by just calling the solution, i.e. solution(p). This returns a numpy array of just the coefficients(c) of the posynomial after substitution, and will raise a 'ValueError' if some of the variables in p were not found in solution.

8.4.7 Freeing Fixed Variables

After creating a Model, it may be useful to "free" a fixed variable and resolve. This can be done using the command del m.substitutions["x"], where m is a Model. An example of how to do this is shown below.

```
from gpkit import Variable, Model
x = Variable("x")
y = Variable("y", 3) # fix value to 3
```

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```
m = Model(x, [x >= 1 + y, y >= 1])
_ = m.solve()  # optimal cost is 4; y appears in sol["constants"]

del m.substitutions["y"]
_ = m.solve()  # optimal cost is 2; y appears in Free Variables
```

Note that del m.substitutions["y"] affects m but not y.key. y.value will still be 3, and if y is used in a new model, it will still carry the value of 3.

8.5 Composite Objectives

Given n posynomial objectives g_i , you can sweep out the problem's Pareto frontier with the composite objective:

```
\begin{array}{l} g_0w_0\prod_{i\neq 0}v_i+g_1w_1\prod_{i\neq 1}v_i+...+g_n\prod_iv_i\\ \\ \text{where } i\in 0...n-1 \text{ and } v_i=1-w_i \text{ and } w_i\in [0,1] \end{array}
```

GPkit has the helper function composite_objective for constructing these.

The normsub argument specifies an expected value for your solution to normalize the different g_i (you can also do this by hand). The feasibility of the problem should not depend on the normalization, but the spacing of the sweep will.

The sweep argument specifies what points between 0 and 1 you wish to sample the weights at. If you want different resolutions or spacings for different weights, the sweeps argument accepts a list of sweep arrays.

CHAPTER 9

Signomial Programming

Signomial programming finds a local solution to a problem of the form:

```
\begin{array}{ll} \text{minimize} & g_0(x) \\ \text{subject to} & f_i(x)=1, & i=1,....,m \\ & g_i(x)-h_i(x)\leq 1, & i=1,....,n \end{array}
```

where each f is monomial while each g and h is a posynomial.

This requires multiple solutions of geometric programs, and so will take longer to solve than an equivalent geometric programming formulation.

In general, when given the choice of which variables to include in the positive-posynomial / g side of the constraint, the modeler should:

- 1. maximize the number of variables in g,
- 2. prioritize variables that are in the objective,
- 3. then prioritize variables that are present in other constraints.

The .localsolve syntax was chosen to emphasize that signomial programming returns a local optimum. For the same reason, calling .solve on an SP will raise an error.

By default, signomial programs are first solved conservatively (by assuming each h is equal only to its constant portion) and then become less conservative on each iteration.

9.1 Example Usage

```
"""Adapted from t_SP in tests/t_geometric_program.py"""
import gpkit

# Decision variables
x = gpkit.Variable('x')
y = gpkit.Variable('y')
```

```
# must enable signomials for subtraction
with gpkit.SignomialsEnabled():
    constraints = [x >= 1-y, y <= 0.1]

# create and solve the SP
m = gpkit.Model(x, constraints)
print m.localsolve(verbosity=0).summary()
assert abs(m.solution(x) - 0.9) < 1e-6</pre>
```

When using the localsolve method, the reltol argument specifies the relative tolerance of the solver: that is, by what percent does the solution have to improve between iterations? If any iteration improves less than that amount, the solver stops and returns its value.

If you wish to start the local optimization at a particular point x_k , however, you may do so by putting that position (a dictionary formatted as you would a substitution) as the xk argument.

9.2 Sequential Geometric Programs

The method of solving local GP approximations of a non-GP compatible model can be generalized, at the cost of the general smoothness and lack of a need for trust regions that SPs guarantee.

For some applications, it is useful to call external codes which may not be GP compatible. Imagine we wished to solve the following optimization problem:

```
minimize y

subject to y \ge \sin(x)

\frac{\pi}{4} \le x \le \frac{\pi}{2}
```

This problem is not GP compatible due to the sin(x) constraint. One approach might be to take the first term of the Taylor expansion of sin(x) and attempt to solve:

```
Cost
----
0.7854
```

```
Free Variables
-----
x: 0.7854
y: 0.7854
```

We can do better, however, by utilizing some built in functionality of GPkit. For simple cases with a single Variable, GPkit looks for externalfn metadata:

```
"Can be found in gpkit/docs/source/examples/external_sp2.py"
import numpy as np
from gpkit import Variable, Model

x = Variable("x")

def y_ext(self, x0):
    "Returns constraints on y derived from x0"
    if x not in x0:
        return self >= x
        return self >= x
        return self >= x/x0[x] * np.sin(x0[x])

y = Variable("y", externalfn=y_ext)

m = Model(y, [np.pi/4 <= x, x <= np.pi/2])
print m.localsolve(verbosity=0).summary()</pre>
```

```
Cost
---
0.7071

Free Variables
----
x: 0.7854
y: 0.7071
```

However, for external functions not intrinsically tied to a single variable it's best to use the full ConstraintSet API, as follows:

Assume we have some external code which is capable of evaluating our incompatible function:

```
"""External function for GPkit to call. Can be found
in gpkit/docs/source/examples/external_function.py"""
import numpy as np

def external_code(x):
    "Returns sin(x)"
    return np.sin(x)
```

Now, we can create a ConstraintSet that allows GPkit to treat the incompatible constraint as though it were a signomial programming constraint:

```
"Can be found in qpkit/docs/source/examples/external_constraint.py"
from gpkit.exceptions import InvalidGPConstraint
from external_function import external_code
class ExternalConstraint(object):
   "Class for external calling"
   varkeys = {}
    def ___init___(self, x, y):
        # We need a GPkit variable defined to return in our constraint. The
        # easiest way to do this is to read in the parameters of interest in
        # the initiation of the class and store them here.
        self.x = x
        self.y = y
    def as_posyslt1(self, _):
        "Ensures this is treated as an SGP constraint"
        raise InvalidGPConstraint ("ExternalConstraint cannot solve as a GP.")
    def as_gpconstr(self, x0, _):
        "Returns locally-approximating GP constraint"
        # Unpacking the GPkit variables
        x = self.x
        y = self.y
        # Creating a default constraint for the first solve
        if not x0:
            return (y >= x)
        # Returns constraint updated with new call to the external code
            # Unpack Design Variables at the current point
            x_star = x0["x"]
            # Call external code
            res = external_code(x_star)
        # Return linearized constraint
        return (y >= res*x/x_star)
```

and replace the incompatible constraint in our GP:

```
x >= np.pi/4.,
]

m = Model(objective, constraints)
print m.localsolve(verbosity=0).summary()
```

```
Cost
----
0.7071

Free Variables
-----
x: 0.7854
y: 0.7071
```

which is the expected result. This method has been generalized to larger problems, such as calling XFOIL and AVL.

If you wish to start the local optimization at a particular point x_0 , however, you may do so by putting that position (a dictionary formatted as you would a substitution) as the $\times 0$ argument

CHAPTER 10

Examples

10.1 iPython Notebook Examples

More examples, including some with in-depth explanations and interactive visualizations, can be seen on nbviewer.

10.2 A Trivial GP

The most trivial GP we can think of: minimize x subject to the constraint $x \ge 1$.

```
"Very simple problem: minimize x while keeping x greater than 1."
from gpkit import Variable, Model

# Decision variable
x = Variable("x")

# Constraint
constraints = [x >= 1]

# Objective (to minimize)
objective = x

# Formulate the Model
m = Model(objective, constraints)

# Solve the Model
sol = m.solve(verbosity=0)

# print selected results
print("Optimal cost: %s" % sol["cost"])
print("Optimal x val: %s" % sol(x))
```

Of course, the optimal value is 1. Output:

```
Optimal cost: 1.000000000039595
Optimal x val: 1.0 dimensionless
```

10.3 Maximizing the Volume of a Box

This example comes from Section 2.4 of the GP tutorial, by S. Boyd et. al.

```
"Maximizes box volume given area and aspect ratio constraints."
from gpkit import Variable, Model
# Parameters
alpha = Variable("alpha", 2, "-", "lower limit, wall aspect ratio")
beta = Variable("beta", 10, "-", "upper limit, wall aspect ratio")
gamma = Variable("gamma", 2, "-", "lower limit, floor aspect ratio")
delta = Variable("delta", 10, "-", "upper limit, floor aspect ratio")
A_wall = Variable("A_{wall}", 200, "m^2", "upper limit, wall area")
A_floor = Variable("A_{floor}", 50, "m^2", "upper limit, floor area")
# Decision variables
h = Variable("h", "m", "height")
w = Variable("w", "m", "width")
d = Variable("d", "m", "depth")
# Constraints
constraints = [A_wall >= 2*h*w + 2*h*d,
                A_floor >= w*d
                h/w >= alpha,
                h/w <= beta,
                d/w >= gamma,
                d/w <= delta]</pre>
# Objective function
objective = 1/V # To maximize V, we minimize its reciprocal
# Formulate the Model
m = Model(objective, constraints)
# Solve the Model and print the results table
print m.solve(verbosity=0).table()
```

The output is

```
A_{floor}: 50 [m**2] upper limit, floor area
A_{wall}: 200 [m**2] upper limit, wall area
   alpha : 2
                       lower limit, wall aspect ratio
    beta : 10
                       upper limit, wall aspect ratio
                       upper limit, floor aspect ratio
   delta: 10
   gamma : 2
                       lower limit, floor aspect ratio
Sensitivities
A_{wall} : -1.5
                    upper limit, wall area
   alpha : +0.5
                    lower limit, wall aspect ratio
   gamma: +0.0003 lower limit, floor aspect ratio
A_{floor}: -5.7e-09 upper limit, floor area
    beta: -1.4e-09 upper limit, wall aspect ratio
   delta: -1.4e-09 upper limit, floor aspect ratio
```

10.4 Water Tank

Say we had a fixed mass of water we wanted to contain within a tank, but also wanted to minimize the cost of the material we had to purchase (i.e. the surface area of the tank):

The output is

```
Cost
----
1.293 [m**2]

Free Variables
-----
A: 1.293 [m**2] Surface Area of the Tank
V: 0.1 [m**3] Volume of the Tank
d: [0.464 0.464 0.464 ] [m] Dimension Vector

Sensitivities (continues on next page)
```

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```
M: +0.67 Mass of Water in the Tank
\rho: -0.67 Density of Water in the Tank
```

10.5 Simple Wing

This example comes from Section 3 of Geometric Programming for Aircraft Design Optimization, by W. Hoburg and P. Abbeel.

```
"Minimizes airplane drag for a simple drag and structure model."
import numpy as np
from gpkit import Variable, Model
pi = np.pi
# Constants
k = Variable("k", 1.2, "-", "form factor")
e = Variable("e", 0.95, "-", "Oswald efficiency factor")
mu = Variable("\mu", 1.78e-5, "kg/m/s", "viscosity of air")
rho = Variable("\rho", 1.23, "kg/m^3", "density of air")
tau = Variable("\\tau", 0.12, "-", "airfoil thickness to chord ratio")
N_ult = Variable("N_{ult}", 3.8, "-", "ultimate load factor")
V_min = Variable("V_{min}", 22, "m/s", "takeoff speed")
C_Lmax = Variable("C_{L,max}", 1.5, "-", "max CL with flaps down")
S_{\text{wet}} = Variable("(<math>\{S\}\{S_{\text{wet}}\})", 2.05, "-", "wetted area ratio")
"Wing Weight Coefficent 1")
W_W_{coeff2} = Variable(W_{w_{coeff2}}), 45.24, Pa, 45.24)
                      "Wing Weight Coefficent 2")
CDA0 = Variable("(CDA0)", 0.031, "m^2", "fuselage drag area")
W_0 = Variable("W_0", 4940.0, "N", "aircraft weight excluding wing")
# Free Variables
D = Variable("D", "N", "total drag force")
A = Variable("A", "-", "aspect ratio")
S = Variable("S", "m^2", "total wing area")
V = Variable("V", "m/s", "cruising speed")
W = Variable("W", "N", "total aircraft weight")
Re = Variable("Re", "-", "Reynold's number")
C_D = Variable("C_D", "-", "Drag coefficient of wing")
C_L = Variable("C_L", "-", "Lift coefficent of wing")
C_f = Variable("C_f", "-", "skin friction coefficient")
W_w = Variable("W_w", "N", "wing weight")
constraints = []
# Drag model
C_D_fuse = CDA0/S
C_D_wpar = k*C_f*S_wetratio
C_D_{ind} = C_L **2/(pi*A*e)
constraints += [C_D >= C_D_fuse + C_D_wpar + C_D_ind]
# Wing weight model
```

```
W_w_{strc} = W_w_{coeff1*(N_ult*A**1.5*(W_0*W*S)**0.5)/tau}
W_w_surf = W_W_coeff2 * S
constraints += [W_w >= W_w_surf + W_w_strc]
# and the rest of the models
constraints += [D >= 0.5*rho*S*C_D*V**2,
                Re \leq (rho/mu) \star V \star (S/A) \star \star 0.5,
                C_f >= 0.074/Re **0.2,
                W \ll 0.5*rho*S*C_L*V**2,
                W \le 0.5*rho*S*C_Lmax*V_min**2,
                 W >= W_0 + W_w
print("SINGLE\n=====")
m = Model(D, constraints)
sol = m.solve(verbosity=0)
print(sol.summary())
print("SWEEP\n=====")
N = 2
sweeps = \{V_{min}: ("sweep", np.linspace(20, 25, N)),
          V: ("sweep", np.linspace(45, 55, N)), }
m.substitutions.update(sweeps)
sweepsol = m.solve(verbosity=0)
print(sweepsol.summary())
```

The output is

```
SINGLE
_____
Cost
303.1 [N]
Free Variables
A : 8.46
                     aspect ratio
C_D : 0.02059
                    Drag coefficient of wing
C_L : 0.4988
                     Lift coefficent of wing
C_{f}: 0.003599
                     skin friction coefficient
D: 303.1 [N] total drag force
Re : 3.675e+06
                     Reynold's number
 S: 16.44 [m**2] total wing area
V: 38.15
               [m/s] cruising speed
 W : 7341
                [N]
                      total aircraft weight
W_w : 2401
               [N]
                    wing weight
Most Sensitive
               W_0 : +1 aircraft weight excluding wing
                e: -0.48 Oswald efficiency factor
(\frac{S}{S_{wet}}) : +0.43 \text{ wetted area ratio}
                k : +0.43 form factor
           V_{\min}: -0.37 takeoff speed
SWEEP
____
```

(continues on next page)

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```
Cost.
[ 338
           294
                   396
                            326
                                      ] [N]
Sweep Variables
    V : [ 45
                   45
                            55
                                     55
                                             [m/s] cruising speed
V_{min} : [ 20
                   25
                            20
                                     25
                                             [m/s] takeoff speed
Free Variables
 A : [ 6.2
              8.84
                        4.77
                                 7.16
                                                  aspect ratio
             0.0196
                         0.0123
C_D : [ 0.0146
                                 0.0157
                                                  Drag coefficient of.
⇔wing
C_L : [ 0.296
               0.463
                         0.198
                                 0.31
                                         1
                                                  Lift coefficent of.
⇔wina
C_f : [ 0.00333     0.00361
                         0.00314
                                 0.00342 ]
                                                  skin friction
D: [ 338
                294
                         396
                                  326
                                      ] [N]
                                                  total drag force
Re: [5.38e+06 3.63e+06 7.24e+06 4.75e+06]
                                                  Reynold's number
 S: [ 18.6 12.1 17.3 11.2 ] [m**2] total wing area
 W: [6.85e+03 6.97e+03 6.4e+03 6.44e+03] [N] total aircraft_
⊶weight
W_w : [ 1.91e+03 2.03e+03 1.46e+03 1.5e+03 ] [N]
                                                  wing weight
Most Sensitive
              W_0 : [+0.92]
                              +0.95
                                       +0.85
                                                +0.85
                                                        ] aircraft.
→weight excluding wing
                                       -1
          V_{\min}: [ -0.82
                                                -0.71
                                                        ] takeoff_
                              -0.41
⇔speed
                V : [ +0.59]
                              +0.25
                                       +0.97
                                                +0.75
                                                        ] cruising
⇔speed
                                                       l wetted area
(\frac{S}{S_{wet}}) : [+0.56]
                              +0.45
                                       +0.63
                                                +0.54
→ratio
               k : [ +0.56
                              +0.45
                                       +0.63
                                                +0.54
                                                       | form factor
```

10.6 Simple Beam

In this example we consider a beam subjected to a uniformly distributed transverse force along its length. The beam has fixed geometry so we are not optimizing its shape, rather we are simply solving a discretization of the Euler-Bernoulli beam bending equations using GP.

```
A simple beam example with fixed geometry. Solves the discretized Euler-Bernoulli beam equations for a constant distributed load """

import numpy as np
from gpkit import Variable, VectorVariable, Model, ureg
from gpkit.small_scripts import mag
```

```
class Beam (Model):
    """Discretization of the Euler beam equations for a distributed load.
    Arguments
    N : int
        Number of finite elements that compose the beam.
    L : float
        [m] Length of beam.
    EI : float
        [N m^2] Elastic modulus times cross-section's area moment of inertia.
    q : float or N-vector of floats
        [N/m] Loading density: can be specified as constants or as an array.
    Upper Unbounded
    w\_tip
    def setup(self, N=4):
        EI = Variable("EI", 1e4, "N*m^2")
        dx = Variable("dx", "m", "Length of an element")
        L = Variable("L", 5, "m", "Overall beam length")
        q = VectorVariable(N, "q", 100*np.ones(N), "N/m",
                           "Distributed load at each point")
        V = VectorVariable(N, "V", "N", "Internal shear")
        V_tip = Variable("V_{tip}", 0, "N", "Tip loading")
        M = VectorVariable(N, "M", "N*m", "Internal moment")
        M_{tip} = Variable("M_{tip})", 0, "N*m", "Tip moment")
        th = VectorVariable(N, "\\theta", "-", "Slope")
        th_base = Variable("\\theta_{base}\", 0, "-\", "Base angle\")
        w = VectorVariable(N, "w", "m", "Displacement")
        w_base = Variable("w_{base}", 0, "m", "Base deflection")
        # below: trapezoidal integration to form a piecewise-linear
                 approximation of loading, shear, and so on
        # shear and moment increase from tip to base (left > right)
        shear\_eq = (V >= V.right + 0.5*dx*(q + q.right))
        shear_eq[-1] = (V[-1] >= V_tip) # tip boundary condition
        moment_eq = (M >= M.right + 0.5*dx*(V + V.right))
        moment\_eq[-1] = (M[-1] >= M\_tip)
        # slope and displacement increase from base to tip (right > left)
        theta eq = (th >= th.left + 0.5*dx*(M + M.left)/EI)
        theta_eq[0] = (th[0] >= th_base) # base boundary condition
        displ_eq = (w >= w.left + 0.5*dx*(th + th.left))
        displ_eq[0] = (w[0] >= w_base)
        # minimize tip displacement (the last w)
        self.cost = self.w\_tip = w[-1]
        return [shear_eq, moment_eq, theta_eq, displ_eq,
                L == (N-1) * dx
b = Beam(N=6, substitutions={"L": 6, "EI": 1.1e4, "q": 110*np.ones(6)})
b.zero_lower_unbounded_variables()
sol = b.solve(verbosity=0)
print sol.summary()
w_gp = sol("w") # deflection along beam
L, EI, q = sol("L"), sol("EI"), sol("q")
                                                                (continues on next page)
```

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```
x = np.linspace(0, mag(L), len(q))*ureg.m # position along beam
q = q[0] # assume uniform loading for the check below
w_{exact} = q/(24.*EI) * x**2 * (x**2 - 4*L*x + 6*L**2) # analytic soln
assert max(abs(w_gp - w_exact)) <= 1.1*ureg.cm</pre>
PLOT = False
if PLOT:
   import matplotlib.pyplot as plt
   x_exact = np.linspace(0, L, 1000)
    w_{exact} = q/(24.*EI) * x_{exact**2} * (x_{exact**2} - 4*L*x_{exact} + 6*L**2)
    plt.plot(x, w_gp, color='red', linestyle='solid', marker='^',
             markersize=8)
   plt.plot(x_exact, w_exact, color='blue', linestyle='dashed')
   plt.xlabel('x [m]')
   plt.ylabel('Deflection [m]')
   plt.axis('equal')
    plt.legend(['GP solution', 'Analytical solution'])
    plt.show()
```

The output is

```
Cost
1.62 [m]
Free Variables
_____
  dx : 1.2
                                                 [m]
                                                     Length of an_
⊶element
   M : [ 1.98e+03 1.27e+03 713
                                  317
                                          ...] [N*m] Internal_
⊶moment
                                          ...] [N] Internal shear
   V : [ 660
                 528
                          396
                                   264
                 0.177 0.285
                                 0.341
                                                      Slope
\theta : [ -
                                          . . . ]
                 0.106 0.384
                                  0.759
  w : [ -
                                          ...] [m] Displacement
Most Sensitive
L : +4
                                            Overall beam length
EI : -1
q: [ +0.0072 +0.042 +0.12 +0.23
                                       ... ] Distributed load at each_
→point
```

By plotting the deflection, we can see that the agreement between the analytical solution and the GP solution is good.

CHAPTER 11

Glossary

For an alphabetical listing of all commands, check out the genindex

11.1 gpkit package

11.1.1 Subpackages

gpkit.constraints package

Submodules

gpkit.constraints.array module

Implements ArrayConstraint

```
\textbf{class} \hspace{0.1cm} \texttt{gpkit.constraints.array.ArrayConstraint} \hspace{0.1cm} (\textit{constraints}, \textit{left}, \textit{oper}, \textit{right})
```

Bases: gpkit.constraints.single_equation.SingleEquationConstraint, gpkit.constraints.set.ConstraintSet

A ConstraintSet for prettier array-constraint printing.

ArrayConstraint gets its *sub* method from ConstrainSet, and so *left* and *right* are only used for printing.

When created by NomialArray *left* and *right* are likely to be either NomialArrays or Varkeys of VectorVariables.

subinplace(subs)

Substitutes in place, updating self.substitutions accordingly.

Keys substituted with *subinplace* are no longer present, so if such a key is also in self.substitutions that substitution is now orphaned. If *subs[key]* describes some key in the

ConstraintSet (i.e. one key has been substituted for another), then a substitution is added, mapping the orphaned value to this new key; otherwise, an error is raised.

gpkit.constraints.bounded module

```
Implements Bounded
class gpkit.constraints.bounded.Bounded(constraints, verbosity=1, eps=1e-30,
                                                       lower=None, upper=None)
     Bases: gpkit.constraints.set.ConstraintSet
     Bounds contained variables so as to ensure dual feasibility.
     constraints [iterable] constraints whose varkeys will be bounded
     substitutions [dict] as in ConstraintSet.__init__
     verbosity [int]
          how detailed of a warning to print 0: nothing 1: print warnings
     eps [float] default lower bound is eps, upper bound is 1/eps
     lower [float] lower bound for all varkeys, replaces eps
     upper [float] upper bound for all varkeys, replaces 1/eps
     check_boundaries (result)
          Creates (and potentially prints) a dictionary of unbounded variables.
     process_result (result)
          Add boundedness to the model's solution
     sens_from_dual(las, nus, result)
          Return sensitivities while capturing the relevant lambdas
gpkit.constraints.bounded.varkey_bounds(varkeys, lower, upper)
     Returns constraints list bounding all varkeys.
     varkeys [iterable] list of varkeys to create bounds for
     lower [float] lower bound for all varkeys
     upper [float] upper bound for all varkeys
gpkit.constraints.costed module
Implement CostedConstraintSet
class gpkit.constraints.costed.CostedConstraintSet (cost, constraints, sub-
                                                                      stitutions=None)
     Bases: gpkit.constraints.set.ConstraintSet
     A ConstraintSet with a cost
     cost: gpkit.Posynomial constraints: Iterable substitutions: dict
     controlpanel (*args, **kwargs)
          Easy model control in IPython / Jupyter
          Like interact(), but with the ability to control sliders and their ranges live. args and kwargs are
          passed on to interact()
```

```
interact (ranges=None, fn_of_sol=None, **solvekwargs)
```

Easy model interaction in IPython / Jupyter

By default, this creates a model with sliders for every constant which prints a new solution table whenever the sliders are changed.

fn_of_sol [function] The function called with the solution after each solve that displays the result. By default prints a table.

ranges [dictionary {str: Slider object or tuple}] Determines which sliders get created. Tuple values may contain two or three floats: two correspond to (min, max), while three correspond to (min, step, max)

**solvekwargs kwargs which get passed to the solve()/localsolve() method.

```
reset_varkeys()
```

Resets varkeys to what is in the cost and constraints

```
rootconstr_latex(excluded=None)
```

Latex showing cost, to be used when this is the top constraint

```
rootconstr str(excluded=None)
```

String showing cost, to be used when this is the top constraint

```
subinplace (subs)
```

Substitutes in place.

gpkit.constraints.gp module

Implement the GeometricProgram class

Bases: gpkit.constraints.costed.CostedConstraintSet, gpkit.nomials.data.NomialData

Standard mathematical representation of a GP.

cost [Constraint] Posynomial to minimize when solving

constraints [list of Posynomials] Constraints to maintain when solving (implicitly Posynomials ≤ 1) GeometricProgram does not accept equality constraints (e.g. x == 1);

instead use two inequality constraints (e.g. $x \le 1$, $1/x \le 1$)

verbosity [int (optional)] If verbosity is greater than zero, warns about missing bounds on creation.

solver_out and solver_log are set during a solve result is set at the end of a solve if solution status is optimal

check_solution (cost, primal, nu, la, tol=0.001, abstol=1e-20)

Run a series of checks to mathematically confirm sol solves this GP

cost: float cost returned by solver

```
primal: list primal solution returned by solver
           nu: numpy.ndarray monomial lagrange multiplier
           la: numpy.ndarray posynomial lagrange multiplier
           RuntimeWarning, if any problems are found
      gen()
           Generates nomial and solve data (A, p_idxs) from posynomials
      solve (solver=None,
                               verbosity=1,
                                               warn_on_check=False,
                                                                         process_result=True,
              **kwargs)
           Solves a GeometricProgram and returns the solution.
           solver [str or function (optional)] By default uses one of the solvers found during installation.
               If set to "mosek", "mosek_cli", or "cvxopt", uses that solver. If set to a function, passes
               that function cs, A, p_idxs, and k.
           verbosity [int (optional)] If greater than 0, prints solver name and solve time.
           **kwargs: Passed to solver constructor and solver function.
           result [dict] A dictionary containing the translated solver result; keys below.
               cost [float] The value of the objective at the solution.
               variables [dict] The value of each variable at the solution.
               sensitivities [dict]
                   monomials [array of floats] Each monomial's dual variable value at the solution.
                   posynomials [array of floats] Each posynomials's dual variable value at the solution.
gpkit.constraints.gp.check_mono_eq_bounds(missingbounds, meq_bounds)
     Bounds variables with monomial equalities
gpkit.constraints.gp.genA(exps, varlocs, meq_idxs)
      Generates A matrix from exps and varidxs
           exps [list of Hashvectors] Exponents for each monomial in a GP
           varidxs [dict] Locations of each variable in exps
           A [sparse Cootmatrix] Exponents of the various free variables for each monomial: rows
               of A are monomials, columns of A are variables.
           missingbounds [dict] Keys: variables that lack bounds. Values: which bounds are
               missed.
gpkit.constraints.model module
Implements Model
class gpkit.constraints.model.Model(cost=None,
                                                                 constraints=None,
                                                                                       *args,
                                                   **kwargs)
      Bases: gpkit.constraints.costed.CostedConstraintSet
     Symbolic representation of an optimization problem.
     The Model class is used both directly to create models with constants and sweeps, and indirectly
```

inherited to create custom model classes.

```
cost [Posynomial (optional)] Defaults to Monomial(1).
constraints [ConstraintSet or list of constraints (optional)] Defaults to an empty list.
substitutions [dict (optional)] This dictionary will be substituted into the problem before solving,
     and also allows the declaration of sweeps and linked sweeps.
name [str (optional)] Allows "naming" a model in a way similar to inherited instances, and over-
     rides the inherited name if there is one.
program is set during a solve solution is set at the end of a solve
as_gpconstr (x0, substitutions=None)
     Returns approximating constraint, keeping name and num
autosweep (sweeps, tol=0.01, samplepoints=100, **solveargs)
     Autosweeps {var: (start, end)} pairs in sweeps to tol.
     Returns swept and sampled solutions. The original simplex tree can be accessed at sol.bst
debug (solver=None, verbosity=1, **solveargs)
     Attempts to diagnose infeasible models.
     If a model debugs but errors in a process_result call, debug again with process_results=False
gp (constants=None, **kwargs)
     Return program version of self
     program: NomialData Class to return, e.g. GeometricProgram or SequentialGeometricPro-
          gram
     return_attr: string attribute to return in addition to the program
localsolve (solver=None, verbosity=1, skipsweepfailures=False, **kwargs)
     Forms a mathematical program and attempts to solve it.
     solver [string or function (optional)] If None, uses the default solver found in installation.
     verbosity [int (optional)] If greater than 0 prints runtime messages. Is decremented by one
          and then passed to programs.
     skipsweepfailures [bool (optional)] If True, when a solve errors during a sweep, skip it.
     **kwargs: Passed to solver
     sol [SolutionArray] See the SolutionArray documentation for details.
     ValueError if the program is invalid. RuntimeWarning if an error occurs in solving or parsing
     the solution.
name = None
naming = None
num = None
program = None
solution = None
solve (solver=None, verbosity=1, skipsweepfailures=False, **kwargs)
     Forms a mathematical program and attempts to solve it.
     solver [string or function (optional)] If None, uses the default solver found in installation.
     verbosity [int (optional)] If greater than 0 prints runtime messages. Is decremented by one
          and then passed to programs.
```

```
skipsweepfailures [bool (optional)] If True, when a solve errors during a sweep, skip it.
          **kwargs : Passed to solver
          sol [SolutionArray] See the SolutionArray documentation for details.
          ValueError if the program is invalid. RuntimeWarning if an error occurs in solving or parsing
          the solution.
     sp (constants=None, **kwargs)
          Return program version of self
          program: NomialData Class to return, e.g. GeometricProgram or SequentialGeometricPro-
          return_attr: string attribute to return in addition to the program
     subconstr_latex (excluded=None)
          The collapsed appearance of a ConstraintBase
     subconstr_str (excluded=None)
          The collapsed appearance of a ConstraintBase
     sweep (sweeps, **solveargs)
          Sweeps {var: values} pairs in sweeps. Returns swept solutions.
     verify_docstring()
          Verifies docstring bounds are sufficient but not excessive.
     zero_lower_unbounded_variables()
          Recursively substitutes 0 for variables that lack a lower bound
gpkit.constraints.model.get_relaxed(relaxvals, mapped_list, min_return=I)
     Determines which relaxvars are considered 'relaxed'
gpkit.constraints.prog_factories module
Scripts for generating, solving and sweeping programs
gpkit.constraints.prog_factories.run_sweep (genfunction,
                                                                           self,
                                                                                   solution,
                                                           skipsweepfailures,
                                                                                 constants,
                                                           sweep, linked, solver, verbosity,
                                                            **kwargs)
     Runs through a sweep.
gpkit.constraints.relax module
Models for assessing primal feasibility
class gpkit.constraints.relax.ConstantsRelaxed(constraints,
                                                                                        in-
                                                                 clude_only=None,
                                                                                        ex-
                                                                 clude=None)
     Bases: gpkit.constraints.set.ConstraintSet
     Relax constraints in a constraintset.
     constraints [iterable] Constraints which will be relaxed (made easier).
     include only [set] if declared, variable names must be on this list to be relaxed
     exclude [set] if declared, variable names on this list will never be relaxed
```

relaxvars [Variable] The variables controlling the relaxation. A solved value of 1 means no relaxation was necessary or optimal for a particular constant. Higher values indicate the amount by which that constant has been made easier: e.g., a value of 1.5 means it was made 50 percent easier in the final solution than in the original problem. Of course, this can also be determined by looking at the constant's new value directly.

```
process_result (result)
```

Does arbitrary computation / manipulation of a program's result

There's no guarantee what order different constraints will process results in, so any changes made to the program's result should be careful not to step on other constraint's toes.

- · check that an inequality was tight
- add values computed from solved variables

```
class gpkit.constraints.relax.ConstraintsRelaxed(constraints)
```

Bases: gpkit.constraints.set.ConstraintSet

Relax constraints, as in Eqn. 11 of [Boyd2007].

constraints [iterable] Constraints which will be relaxed (made easier).

relaxvars [Variable] The variables controlling the relaxation. A solved value of 1 means no relaxation was necessary or optimal for a particular constraint. Higher values indicate the amount by which that constraint has been made easier: e.g., a value of 1.5 means it was made 50 percent easier in the final solution than in the original problem.

[Boyd2007]: "A tutorial on geometric programming", Optim Eng 8:67-122

```
class gpkit.constraints.relax.ConstraintsRelaxedEqually(constraints)
    Bases: gpkit.constraints.set.ConstraintSet
```

Relax constraints the same amount, as in Eqn. 10 of [Boyd2007].

constraints [iterable] Constraints which will be relaxed (made easier).

relaxvar [Variable] The variable controlling the relaxation. A solved value of 1 means no relaxation. Higher values indicate the amount by which all constraints have been made easier: e.g., a value of 1.5 means all constraints were 50 percent easier in the final solution than in the original problem.

[Boyd2007]: "A tutorial on geometric programming", Optim Eng 8:67-122

gpkit.constraints.set module

```
Implements ConstraintSet
```

```
class gpkit.constraints.set.ConstraintSet(constraints, substitutions=None)
    Bases: list
```

Recursive container for ConstraintSets and Inequalities

```
append (value)
```

L.append(object) – append object to end

```
as_gpconstr(x0, substitutions=None)
```

Returns GPConstraint approximating this constraint at x0

When x0 is none, may return a default guess.

as_posyslt1 (substitutions=None)

Returns list of posynomials which must be kept <= 1

flat (constraintsets=True)

Yields contained constraints, optionally including constraintsets.

latex (excluded=None)

LaTeX representation of a ConstraintSet.

process result (result)

Does arbitrary computation / manipulation of a program's result

There's no guarantee what order different constraints will process results in, so any changes made to the program's result should be careful not to step on other constraint's toes.

- · check that an inequality was tight
- · add values computed from solved variables

reset_varkeys()

Goes through constraints and collects their varkeys.

rootconstr_latex (excluded=None)

The appearance of a ConstraintSet in addition to its contents

rootconstr_str (excluded=None)

The appearance of a ConstraintSet in addition to its contents

sens_from_dual(las, nus, result)

Computes constraint and variable sensitivities from dual solution

las [list] Sensitivity of each posynomial returned by self.as_posyslt1

nus: list of lists Each posynomial's monomial sensitivities

constraint_sens [dict] The interesting and computable sensitivities of this constraint

var_senss [dict] The variable sensitivities of this constraint

str_without (excluded=None)

String representation of a ConstraintSet.

subconstr_latex(excluded=None)

The collapsed appearance of a ConstraintSet

subconstr_str (excluded=None)

The collapsed appearance of a ConstraintSet

subinplace (subs)

Substitutes in place, updating self.substitutions accordingly.

Keys substituted with *subinplace* are no longer present, so if such a key is also in self.substitutions that substitution is now orphaned. If *subs[key]* describes some key in the ConstraintSet (i.e. one key has been substituted for another), then a substitution is added, mapping the orphaned value to this new key; otherwise, an error is raised.

topvar (key)

If a variable by a given name exists in the top model, return it

unique_varkeys = frozenset([])

variables_byname (key)

Get all variables with a given name

varkeys = None

```
gpkit.constraints.set.add_meq_bounds (bounded, meq_bounded)
    Iterates through meq_bounds until convergence
gpkit.constraints.set.raise_badelement (cns, i, constraint)
    Identify the bad element and raise a ValueError
gpkit.constraints.set.raise_elementhasnumpybools (constraint)
    Identify the bad subconstraint array and raise a ValueError
```

gpkit.constraints.sgp module

Implement the SequentialGeometricProgram class

Bases: gpkit.constraints.costed.CostedConstraintSet

Prepares a collection of signomials for a SP solve.

cost [Posynomial] Objective to minimize when solving

constraints [list of Constraint or SignomialConstraint objects] Constraints to maintain when solving (implicitly Signomials <= 1)

verbosity [int (optional)] Currently has no effect: SequentialGeometricPrograms don't know anything new after being created, unlike GeometricPrograms.

gps is set during a solve result is set at the end of a solve

gp(x0=None, mutategp=False)

The GP approximation of this SP at x0.

init_gp (substitutions, x0=None)

Generates a simplified GP representation for later modification

```
localsolve (solver=None, verbosity=1, x0=None, reltol=0.0001, iteration_limit=50, mutategp=True, **kwargs)
```

Locally solves a SequentialGeometricProgram and returns the solution.

solver [str or function (optional)] By default uses one of the solvers found during installation. If set to "mosek", "mosek_cli", or "cvxopt", uses that solver. If set to a function, passes that function cs, A, p_idxs, and k.

verbosity [int (optional)] If greater than 0, prints solve time and number of iterations. Each GP is created and solved with verbosity one less than this, so if greater than 1, prints solver name and time for each GP.

x0 [dict (optional)] Initial location to approximate signomials about.

```
reltol [float] Iteration ends when this is greater than the distance between two consecutive solve's objective values.
```

iteration_limit [int] Maximum GP iterations allowed.

*args, **kwargs: Passed to solver function.

result [dict] A dictionary containing the translated solver result.

gpkit.constraints.sigeq module

```
Implements SignomialEquality
```

A constraint of the general form posynomial == posynomial

gpkit.constraints.single_equation module

```
Implements SingleEquationConstraint
```

```
class gpkit.constraints.single\_equation.SingleEquationConstraint(<math>left,
                                                                                   right)
     Bases: object
     Constraint expressible in a single equation.
     func_opers = {'<=': <built-in function le>, '=': <built-in function eq>, '>=':
     latex (excluded=None)
          Latex representation without attributes in excluded list
     latex_opers = {'<=': '\leq', '=': '=', '>=': '\\geq'}
     process_result (result)
          Process solver results
     str_without (excluded=None)
          String representation without attributes in excluded list
     sub (subs)
          Returns a substituted version of this constraint.
     subconstr_latex(excluded)
          The collapsed latex of a constraint
     subconstr_str(excluded)
          The collapsed string of a constraint
gpkit.constraints.single_equation.trycall(obj, attr, arg, default)
     Try to call method of an object, returning default if it does not exist
```

gpkit.constraints.tight module

Implements Tight

```
class gpkit.constraints.tight.Tight (constraints,
                                                              substitutions=None,
                                                                                    rel-
                                               tol=None, raiseerror=False)
     Bases: gpkit.constraints.set.ConstraintSet
     ConstraintSet whose inequalities must result in an equality.
     process_result (result)
          Checks that all constraints are satisfied with equality
     reltol = 1e-06
Module contents
Contains ConstraintSet and related classes and objects
gpkit.interactive package
Submodules
gpkit.interactive.chartjs module
gpkit.interactive.plot sweep module
Implements plot_sweep1d function
gpkit.interactive.plot_sweep.assign_axes (var, posys, axes)
     Assigns axes to posys, creating and formatting if necessary
gpkit.interactive.plot_sweep.format_and_label_axes (var, posys, axes, yla-
                                                                   bel=True)
     Formats and labels axes
gpkit.interactive.plot_sweep.plot_1dsweepgrid(model,
                                                                       sweeps,
                                                                                  posvs.
                                                             origsol=None,
                                                                               tol = 0.01,
                                                             **solveargs)
     Creates and plots a sweep from an existing model
     Example usage: f, _ = plot_sweep_1d(m, {'x': np.linspace(1, 2, 5)}, 'y') f.savefig('mysweep.png')
gpkit.interactive.plotting module
Plotting methods
gpkit.interactive.plotting.compare (models, sweeps, posys, tol=0.001)
     Compares the values of posys over a sweep of several models.
     If posys is of the same length as models, this will plot different variables from different models.
     Currently only supports a single sweepvar.
     Example Usage: compare([aec, fbc], {"R": (160, 300)},
          ["cost", ("W_{rm batt}", "W_{rm fuel}")], tol=0.001)
gpkit.interactive.plotting.plot_convergence (model)
     Plots the convergence of a signomial programming model
     model: Model Signomial programming model that has already been solved
```

matplotlib.pyplot Figure Plot of cost as functions of SP iteration

gpkit.interactive.ractor module

```
Implements Ractor-based interactive CADtoons
```

showtables=('cost', 'sensitivities'))

Creates interactive iPython widget for controlling a CADtoon

gpkit.interactive.sankey module

gpkit.interactive.widgets module

Module contents

Module for the interactive and plotting functions of GPkit

gpkit.nomials package

Submodules

gpkit.nomials.array module

Module for creating NomialArray instances.

Example

```
>>> x = gpkit.Monomial('x')
>>> px = gpkit.NomialArray([1, x, x**2])
```

```
class gpkit.nomials.array.NomialArray
```

Bases: numpy.ndarray

A Numpy array with elementwise inequalities and substitutions.

input_array: array-like

```
>>> px = gpkit.NomialArray([1, x, x**2])
```

```
latex (matwrap=True)
```

Returns 1D latex list of contents.

left

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Returns $(0, self[0], self[1] \dots self[N-1])$

```
Returns the array and argument's outer product.
     padleft (padding)
          Returns ({padding}, self[0], self[1] ... self[N])
     padright (padding)
          Returns (self[0], self[1] ... self[N], {padding})
     prod (*args, **kwargs)
          Returns a product. O(N) if no arguments and only contains monomials.
     right
          Returns (self[1], self[2] ... self[N], 0)
     str_without (excluded=None)
           Returns string without certain fields (such as 'models').
     sub (subs, require_positive=True)
          Substitutes into the array
     sum(*args, **kwargs)
          Returns a sum. O(N) if no arguments are given.
     units
          units must have same dimensions across the entire nomial array
     vectorize(function, *args, **kwargs)
          Apply a function to each terminal constraint, returning the array
gpkit.nomials.array.array_constraint(symbol, func)
     Return function which creates constraints of the given operator.
gpkit.nomials.core module
The shared non-mathematical backbone of all Nomials
class gpkit.nomials.core.Nomial(hmap)
     Bases: gpkit.nomials.data.NomialData
     Shared non-mathematical properties of all nomials
     latex (excluded=None)
          Latex representation, parsing excluded just as .str_without does
     prod()
          Return self for compatibility with NomialArray
     str without (excluded=None)
          String representation, excluding fields ('units', varkey attributes)
     sub = None
     sum()
          Return self for compatibility with NomialArray
     to (units)
          Create new Signomial converted to new units
     unitstr(into='%s', options='~', dimless=")
          Returns the string corresponding to an object's units.
```

outer (other)

value

Self, with values substituted for variables that have values

float, if no symbolic variables remain after substitution (Monomial, Posynomial, or Nomial), otherwise.

gpkit.nomials.data module

Machinery for exps, cs, varlocs data – common to nomials and programs

```
class gpkit.nomials.data.NomialData(hmap)
    Bases: object
```

Object for holding cs, exps, and other basic 'nomial' properties.

cs: array (coefficient of each monomial term) exps: tuple of {VarKey: float} (exponents of each monomial term) varlocs: {VarKey: list} (terms each variable appears in) units: pint.UnitsContainer

cs

Create cs or return cached cs

diff(var)

Derivative of this with respect to a Variable

var (Variable): Variable to take derivative with respect to

NomialData

exps

Create exps or return cached exps

values

The NomialData's values, created when necessary.

varkevs

The NomialData's varkeys, created when necessary for a substitution.

varlocs

Create varlocs or return cached varlocs

gpkit.nomials.map module

Implements the NomialMap class

```
class gpkit.nomials.map.NomialMap
    Bases: gpkit.small_classes.HashVector
```

Class for efficent algebraic represention of a nomial

A NomialMap is a mapping between hashvectors representing exponents and their coefficients in a posynomial.

For example, $\{\{x:1\}: 2.0, \{y:1\}: 3.0\}$ represents 2*x + 3*y, where x and y are VarKey objects.

```
csmap = None
```

diff(varkey)

Differentiates a NomialMap with respect to a varkey

```
expmap = None
```

```
mmap (orig)
          Maps substituted monomials back to the original nomial
          self.expmap is the map from pre- to post-substitution exponents, and takes
                                                                                             form
               {original_exp: new_exp}
          self.csmap is the map from pre-substitution exponents to coefficients.
          m_from_ms is of the form {new_exp: [old_exps, ]}
          pmap is of the form [{orig_idx1: fraction1, orig_idx2: fraction2, },] where at the index
               corresponding to each new_exp is a dictionary mapping the indices corresponding to
               the old exps to their fraction of the post-substitution coefficient
      remove zeros()
          Removes zeroed exponents and monomials.
          If only_check_cs is True, checks only whether any values are zero. If False also checks
          whether any exponents in the keys are zero.
      sub (substitutions, varkeys, parsedsubs=False)
          Applies substitutions to a NomialMap
          substitutions [(dict-like)] list of substitutions to perform
          varkeys [(set-like)] varkeys that are present in self (required argument so as to require effi-
               cient code)
          parsedsubs [bool] flag if the substitutions have already been parsed to contain only keys in
               varkeys
     to (units)
          Returns a new NomialMap of the given units
     units = None
     units_of_product (thing, thing2=None)
          Sets units to those of thing*thing2
qpkit.nomials.map.subinplace(cp, exp, o_exp, vk, cval, expval, exps_covered)
     Modifies cp by substituing cval/expval for vk in exp
gpkit.nomials.math module
Signomial, Posynomial, Monomial, Constraint, & MonoEQCOnstraint classes
class qpkit.nomials.math.Monomial(hmap=None, cs=1, require positive=True,
                                                **descr)
      Bases: gpkit.nomials.math.Posynomial
     A Posynomial with only one term
     Same as Signomial. Note: Monomial historically supported several different init formats
          These will be deprecated in the future, replaced with a single __init__ syntax, same as
          Signomial.
      C
          Creates c or returns a cached c
      exp
          Creates exp or returns a cached exp
```

```
mono approximation (x0)
          Monomial approximation about a point x0
          x0 (dict): point to monomialize about
          Monomial (unless self(x0) < 0, in which case a Signomial is returned)
class qpkit.nomials.math.MonomialEquality(left, oper, right)
     Bases: gpkit.nomials.math.PosynomialInequality
     A Constraint of the form Monomial == Monomial.
     as_posyslt1 (substitutions=None)
          Tags posynomials for dual feasibility checking
     sens_from_dual(la, nu, result)
          Returns the variable/constraint sensitivities from lambda/nu
class gpkit.nomials.math.Posynomial(hmap=None, cs=1, require_positive=True,
                                                **descr)
     Bases: gpkit.nomials.math.Signomial
     A Signomial with strictly positive cs
     Same as Signomial. Note: Posynomial historically supported several different init formats
          These will be deprecated in the future, replaced with a single __init__ syntax, same as
          Signomial.
     mono lower bound (x0)
          Monomial lower bound at a point x0
          x0 (dict): point to make lower bound exact
          Monomial
class gpkit.nomials.math.PosynomialInequality(left, oper, right)
     Bases: gpkit.nomials.math.ScalarSingleEquationConstraint
     A constraint of the general form monomial >= posynomial Stored in the posylt1_rep attribute as a
     single Posynomial (self \leq 1) Usually initialized via operator overloading, e.g. cc = (y^{**2} > 1 + 1)
     x)
     as_gpconstr(x0, substitutions)
          The GP version of a Posynomial constraint is itself
     as_posyslt1 (substitutions=None)
          Returns the posys <= 1 representation of this constraint.
     sens_from_dual(la, nu, result)
          Returns the variable/constraint sensitivities from lambda/nu
class gpkit.nomials.math.ScalarSingleEquationConstraint (left,
                                                                                  oper.
                                                                          right)
     Bases: gpkit.constraints.single_equation.SingleEquationConstraint
     A SingleEquationConstraint with scalar left and right sides.
     nomials = []
     subinplace (substitutions)
          Modifies the constraint in place with substitutions.
class qpkit.nomials.math.Signomial(hmap=None, cs=1, require positive=True,
                                              **descr)
     Bases: gpkit.nomials.core.Nomial
```

A representation of a Signomial.

exps: tuple of dicts Exponent dicts for each monomial term

cs: tuple Coefficient values for each monomial term

require_positive: bool If True and Signomials not enabled, $c \le 0$ will raise ValueError

Signomial Posynomial (if the input has only positive cs) Monomial (if the input has one term and only positive cs)

diff(var)

Derivative of this with respect to a Variable

var (Variable): Variable to take derivative with respect to

Signomial (or Posynomial or Monomial)

$mono_approximation(x0)$

Monomial approximation about a point x0

x0 (**dict**): point to monomialize about

Monomial (unless self(x0) < 0, in which case a Signomial is returned)

posy_negy()

Get the positive and negative parts, both as Posynomials

Posynomial: p_pos and p_neg in (self = p_pos - p_neg) decomposition,

sub (substitutions, require_positive=True)

Returns a nomial with substitued values.

```
3 == (x**2 + y).sub({(x': 1, y: 2)}) == (x).gp.sub(x, 3)
```

substitutions [dict or key] Either a dictionary whose keys are strings, Variables, or VarKeys, and whose values are numbers, or a string, Variable or Varkey.

val [number (optional)] If the substitutions entry is a single key, val holds the value

require_positive [boolean (optional, default is True)] Controls whether the returned value can be a Signomial.

Returns substituted nomial.

subinplace (substitutions)

Substitutes in place.

```
{\tt class} \ {\tt gpkit.nomials.math.SignomialInequality} \ ({\it left, oper, right})
```

Bases: gpkit.nomials.math.ScalarSingleEquationConstraint

A constraint of the general form posynomial >= posynomial Stored internally (exps, cs) as a single Signomial (0 >= self) Usually initialized via operator overloading, e.g. $cc = (y^{**}2 >= 1 + x - y)$ Additionally retains input format (lhs vs rhs) in self.left and self.right Form is self.left >= self.right.

$as_approxsgt(x0)$

Returns monomial-greater-than sides, to be called after as_approxlt1

as_approxslt()

Returns posynomial-less-than sides of a signomial constraint

as_gpconstr(x0, substitutions=None)

Returns GP approximation of an SP constraint at x0

```
as_posyslt1 (substitutions=None)
          Returns the posys <= 1 representation of this constraint.
class gpkit.nomials.math.SingleSignomialEquality(left, right)
     Bases: gpkit.nomials.math.SignomialInequality
     A constraint of the general form posynomial == posynomial
     as\_approxsgt(x\theta)
          Returns monomial-greater-than sides, to be called after as_approxlt1
     as_approxslt()
          Returns posynomial-less-than sides of a signomial constraint
     as_gpconstr (x0, substitutions=None)
          Returns GP approximation of an SP constraint at x0
     as_posyslt1 (substitutions=None)
          Returns the posys <= 1 representation of this constraint.
gpkit.nomials.substitution module
Scripts to parse and collate substitutions
gpkit.nomials.substitution.append_sub(sub, keys, constants, sweep, linkedsweep)
     Appends sub to constants, sweep, or linkedsweep.
gpkit.nomials.substitution.parse_subs (varkeys, substitutions, clean=False)
     Seperates subs into constants, sweeps linkedsweeps actually present.
gpkit.nomials.variables module
Implement Variable and Array Variable classes
class gpkit.nomials.variables.ArrayVariable
     Bases: gpkit.nomials.array.NomialArray
     A described vector of singlet Monomials.
     shape [int or tuple] length or shape of resulting array
     *args:
          may contain "name" (Strings)
                  "value" (Iterable) "units" (Strings + Quantity)
              and/or "label" (Strings)
     **descr: VarKey description
     NomialArray of Monomials, each containing a VarKey with name '$name_{i}', where $name is
     the vector's name and i is the VarKey's index.
class gpkit.nomials.variables.Variable(*args, **descr)
     Bases: gpkit.nomials.math.Monomial
     A described singlet Monomial.
     *args [list]
          may contain "name" (Strings)
```

```
"value" (Numbers + Quantity) or (Iterable) for a sweep "units" (Strings + Quan-
                   tity)
               and/or "label" (Strings)
     **descr [dict] VarKey description
     Monomials containing a VarKey with the name '$name', where $name is the vector's name and i is
     the VarKey's index.
     sub (*args, **kwargs)
          Same as nomial substitution, but also allows single-argument calls
          x = Variable('x') assert x.sub(3) == Variable('x', value=3)
     to (units)
          Create new Signomial converted to new units
class gpkit.nomials.variables.VectorizableVariable(*args, **descr)
               qpkit.nomials.variables.Variable, qpkit.nomials.variables.
     ArrayVariable
     A Variable outside a vectorized environment, an Array Variable within.
gpkit.nomials.variables.veclinkedfn(linkedfn,i)
     Generate an indexed linking function.
Module contents
Contains nomials, inequalities, and arrays
gpkit.tools package
Submodules
gpkit.tools.autosweep module
Tools for optimal fits to GP sweeps
class gpkit.tools.autosweep.BinarySweepTree (bounds, sols, sweptvar, cost-
                                                            posy)
     Bases: object
     Spans a line segment. May contain two subtrees that divide the segment.
     bounds [two-element list] The left and right boundaries of the segment
     sols [two-element list] The left and right solutions of the segment
     costs [array] The left and right logcosts of the segment
     splits [None or two-element list] If not None, contains the left and right subtrees
     splitval [None or float] The worst-error point, where the split will be if tolerance is too low
     splitlb [None or float] The cost lower bound at splitval
```

splitub [None or float] The cost upper bound at splitval

Creates subtrees from bounds[0] to splitval and splitval to bounds[1]

add split(splitval, splitsol)

```
add_splitcost (splitval, splitlb, splitub)
          Adds a splitval, lower bound, and upper bound
     cost_at (_, value, bound=None)
          Logspace interpolates between split and costs. Guaranteed bounded.
     min bst (value)
          Returns smallest bst around value.
     posy_at (posy, value)
          Logspace interpolates between sols to get posynomial values.
          No guarantees, just like a regular sweep.
     sample_at (values)
          Creates a SolutionOracle at a given range of values
     solarray
          Returns a solution array of all the solutions in an autosweep
     sollist
          Returns a list of all the solutions in an autosweep
class gpkit.tools.autosweep.SolutionOracle(bst, sampled_at)
     Bases: object
     Acts like a SolutionArray for autosweeps
     cost lb()
          Gets cost lower bounds from the BST and units them
     cost_ub()
          Gets cost upper bounds from the BST and units them
     plot (posys=None, axes=None)
          Plots the sweep for each posy
     solarray
          Returns a solution array of all the solutions in an autosweep
gpkit.tools.autosweep_1d(model, logtol, sweepvar, bounds, **solvek-
                                               wargs)
     Autosweep a model over one sweepvar
gpkit.tools.autosweep.get_tol(costs, bounds, sols, variable)
     Gets the intersection point and corresponding bounds from two solutions.
gpkit.tools.autosweep.recurse_splits (model, bst, variable, logtol, solvekwargs,
     Recursively splits a BST until logtol is reached
gpkit.tools.docstring module
Docstring-parsing methods
gpkit.tools.docstring.check_and_parse_flag(string,
                                                                      flag,
                                                                                 declara-
                                                          tion_func=None)
     Checks for instances of flag in string and parses them.
gpkit.tools.docstring.constant_declare(string, flag, idx2, countstr)
     Turns Variable declarations into Constant ones
```

```
gpkit.tools.docstring.expected_unbounded(instance, doc)
     Gets expected-unbounded variables from a string
gpkit.tools.docstring.parse_variables(string)
     Parses a string to determine what variables to create from it
gpkit.tools.docstring.variable_declaration(nameval, units, label, line)
     Turns parsed output into a Variable declaration
gpkit.tools.docstring.vv_declare(string, flag, idx2, countstr)
     Turns Variable declarations into VectorVariable ones
gpkit.tools.fmincon module
A module to facilitate testing GPkit against fmincon
gpkit.tools.fmincon.generate_mfiles(model,
                                                                          logspace=False,
                                                algorithm='interior-point',
                                                                            guess='ones',
                                                                gradconstr='on',
                                                gradobi='on',
                                                files=True)
     A method for preparing fmincon input files to run a GPkit program
     INPUTS: model [GPkit model] The model to replicate in fmincon
          logspace [Boolean] Whether to re-produce the model in logspace
          algorithm: [string] Algorithm used by fmincon 'interior-point': uses the interior point
              solver 'SQP': uses the sequential quadratic programming solver
          guess: [string] The type of initial guess used 'ones': One for each variable 'order-of-
              magnitude-floor': The "log-floor" order of
                  magnitude of the GP/SP optimal solution (i.e. O(99)=10)
              'order-of-magnitude-round': The "log-nearest" order of magnitude of the GP/SP
                  optimal solution (i.e. O(42)=100)
              'almost-exact-solution': The GP/SP optimal solution rounded to 1 significant figure
              OR [list] The actual values of initial guess to use
          gradconstr: [string] Include analytical constraint gradients? 'on': Yes 'off': No
          gradobj: [string] Include analytical objective gradients? 'on': Yes 'off': No
          writefiles: [Boolean] whether or not to actually write the m files
gpkit.tools.fmincon.make_initial_guess(model,
                                                                newlist,
                                                                            guess='ones',
                                                    logspace=False)
     Returns initial guess
gpkit.tools.spdata module
Implements SPData class
class gpkit.tools.spdata.SPData(model)
     Bases: gpkit.nomials.data.NomialData
     Generates matrices describing an SP.
```

```
>>> spdata = SPData(m)
>>> spdata.save('example_sp.h5')
```

save (filename)

Save spdata to an h5 file.

gpkit.tools.tools module

```
Non-application-specific convenience methods for GPkit
gpkit.tools.tools.composite_objective(*objectives, **kwargs)
     Creates a cost function that sweeps between multiple objectives.
gpkit.tools.tools.mdmake(filename, make_tex=True)
     Make a python file and (optional) a pandoc-ready .tex.md file
gpkit.tools.tools.mdparse(filename, return_tex=False)
     Parse markdown file, returning as strings python and (optionally) .tex.md
gpkit.tools.tools.te_exp_minus1(posy, nterm)
     Taylor expansion of e^{posy} - 1
     posy [gpkit.Posynomial] Variable or expression to exponentiate
     nterm [int] Number of non-constant terms in resulting Taylor expansion
     gpkit.Posynomial Taylor expansion of e^{posy} - 1, carried to nterm terms
gpkit.tools.tools.te_secant(var, nterm)
     Taylor expansion of secant(var).
     var [gpkit.monomial] Variable or expression argument
     nterm [int] Number of non-constant terms in resulting Taylor expansion
     gpkit.Posynomial Taylor expansion of secant(x), carried to nterm terms
gpkit.tools.tools.te_tangent(var, nterm)
     Taylor expansion of tangent(var).
     var [gpkit.monomial] Variable or expression argument
     nterm [int] Number of non-constant terms in resulting Taylor expansion
     gpkit.Posynomial Taylor expansion of tangent(x), carried to nterm terms
```

Module contents

Contains miscellaneous tools including fmincon comparison tool

11.1.2 Submodules

11.1.3 gpkit.build module

Finds solvers, sets gpkit settings, and builds gpkit

```
class gpkit.build.CVXopt
    Bases: gpkit.build.SolverBackend
    CVXopt finder.
     look()
         Attempts to import cvxopt.
    name = 'cvxopt'
class gpkit.build.Mosek
    Bases: gpkit.build.SolverBackend
    MOSEK finder and builder.
    bin dir = None
    build()
         Builds a dynamic library to GPKITBUILD or $HOME/.gpkit
    expopt_files = None
    flags = None
    lib name = None
    lib_path = None
    look()
         Looks in default install locations for latest mosek version.
    name = 'mosek'
    patches = {'dgopt.c': {'printf("Number of Hessian non-zeros: %d\\n",nlh[0]->numh
    version = None
class gpkit.build.MosekCLI
    Bases: gpkit.build.SolverBackend
    MOSEK command line interface finder.
    look()
         Attempts to run mskexpopt.
    name = 'mosek_cli'
class gpkit.build.SolverBackend
    Bases: object
    Inheritable class for finding solvers. Logs.
    build = None
    installed = False
    look = None
    name = None
gpkit.build.build_gpkit()
    Builds GPkit
gpkit.build.call(cmd)
    Calls subprocess. Logs.
gpkit.build.diff (filename, diff_dict)
     Applies a simple diff to a file. Logs.
```

```
gpkit.build.isfile(path)
     Returns true if there's a file at $path. Logs.
gpkit.build.log(*args)
     Print a line and append it to the log string.
gpkit.build.pathjoin(*args)
     Join paths, collating multiple arguments.
gpkit.build.rebuild()
     Changes to the installed gpkit directory and runs build_gpkit()
gpkit.build.replacedir(path)
     Replaces directory at $path. Logs.
```

11.1.4 gpkit.exceptions module

GPkit-specific Exception classes

```
exception gpkit.exceptions.InvalidGPConstraint
    Bases: exceptions. Exception
```

Raised when a non-GP-compatible constraint is used in a GP

11.1.5 gpkit.globals module

```
global mutable variables
```

Bases: object

```
class gpkit.globals.NamedVariables (model)
    Bases: object
```

Creates an environment in which all variables have a model name and num appended to their varkeys.

```
class gpkit.globals.SignomialsEnabled
    Bases: object
```

Class to put up and tear down signomial support in an instance of GPkit.

```
>>> import gpkit
>>> x = gpkit.Variable("x")
>>> y = gpkit.Variable("y", 0.1)
>>> with SignomialsEnabled():
       constraints = [x >= 1-v]
>>> gpkit.Model(x, constraints).localsolve()
```

```
class gpkit.globals.Vectorize(dimension_length)
```

```
Creates an environment in which all variables are exended in an additional dimension.
```

gpkit.globals.begin_variable_naming(model)

```
Appends a model name and num to the environment.
```

```
gpkit.globals.end_variable_naming()
    Pops a model name and num from the environment.
gpkit.globals.load_settings(path=None)
```

Load the settings file at SETTINGS_PATH; return settings dict

11.1.6 gpkit.keydict module

Implements KeyDict and KeySet classes

```
class gpkit.keydict.KeyDict(*args, **kwargs)
Bases; dict
```

KeyDicts do two things over a dict: map keys and collapse arrays.

```
>>> kd = gpkit.keydict.KeyDict()
```

If .keymapping is True, a KeyDict keeps an internal list of VarKeys as canonical keys, and their values can be accessed with any object whose *key* attribute matches one of those VarKeys, or with strings matching any of the multiple possible string interpretations of each key:

For example, after creating the KeyDict kd and setting kd[x] = v (where x is a Variable or VarKey), v can be accessed with by the following keys:

- X
- x.key
- x.name (a string)
- "x_modelname" (x's name including modelname)

Note that if a item is set using a key that does not have a .key attribute, that key can be set and accessed normally.

If .collapse_arrays is True then VarKeys which have a *shape* parameter (indicating they are part of an array) are stored as numpy arrays, and automatically de-indexed when a matching VarKey with a particular *idx* parameter is used as a key.

See also: gpkit/tests/t_keydict.py.

```
collapse arrays = True
     \operatorname{\mathsf{qet}}(k[,d]) \to \operatorname{\mathsf{D}}[k] if k in D, else d. d defaults to None.
     keymapping = True
     parse_and_index (key)
           Returns key if key had one, and veckey/idx for indexed veckeys.
     update (*args, **kwargs)
           Iterates through the dictionary created by args and kwargs
     update_keymap()
           Updates the keymap with the keys in _unmapped_keys
class gpkit.keydict.KeySet(*args, **kwargs)
     Bases: gpkit.keydict.KeyDict
     KeyDicts that don't collapse arrays or store values.
      add (item)
           Adds an item to the keyset
     collapse_arrays = False
     update(*args, **kwargs)
           Iterates through the dictionary created by args and kwargs
gpkit.keydict.clean value(key, value)
     Gets the value of variable-less monomials, so that x.sub(\{x: gpkit.units.m\}) and x.sub(\{x: gp-inits.m\})
```

kit.ureg.m}) are equivalent.

Also converts any quantities to the key's units, because quantities can't/shouldn't be stored as elements of numpy arrays.

11.1.7 gpkit.repr_conventions module

```
Repository for representation standards
```

```
gpkit.repr_conventions.unitstr(units, into='%s', options='~', dimless=")
Returns the string corresponding to an object's units.
```

11.1.8 gpkit.small_classes module

```
Miscellaneous small classes
```

```
class gpkit.small_classes.CootMatrix(row, col, data)
     Bases: object
     A very simple sparse matrix representation.
     append (row, col, data)
          Appends entry to matrix.
     dot (arg)
          Returns dot product with arg.
     tocoo()
          Converts to another type of matrix.
     tocsc()
          Converts to another type of matrix.
     tocsr()
          Converts to a Scipy sparse csr_matrix
     todense()
          Converts to another type of matrix.
     todia()
          Converts to another type of matrix.
     todok()
          Converts to another type of matrix.
class gpkit.small_classes.Count
     Bases: object
     Like python 2's itertools.count, for Python 3 compatibility.
     next()
          Increment self.count and return it
class gpkit.small_classes.DictOfLists
     Bases: dict
     A hierarchy of dicionaries, with lists at the bottom.
          Appends a dict (of dicts) of lists to all held lists.
     atindex(i)
```

Indexes into each list independently.

```
classify (cls)
          Converts dictionaries whose first key isn't a string to given class.
     to_arrays()
          Converts all lists into array.
class gpkit.small_classes.HashVector
     Bases: dict
     A simple, sparse, string-indexed vector. Inherits from dict.
     The HashVector class supports element-wise arithmetic: any undeclared variables are assumed to
     have a value of zero.
     arg: iterable
     >>> x = gpkit.nomials.Monomial('x')
     >>> exp = gpkit.small_classes.HashVector({x: 2})
     copy()
          Return a copy of this
class gpkit.small_classes.SolverLog(verbosity=0, output=None, **kwargs)
     Bases: list
     Adds a write method to list so it's file-like and can replace stdout.
     write(writ)
          Append and potentially write the new line.
gpkit.small_classes.matrix_converter(name)
     Generates conversion function.
11.1.9 gpkit.small scripts module
Assorted helper methods
gpkit.small_scripts.is_sweepvar(sub)
     Determines if a given substitution indicates a sweep.
gpkit.small_scripts.latex_num(c)
     Returns latex string of numbers, potentially using exponential notation.
gpkit.small\_scripts.mag(c)
     Return magnitude of a Number or Quantity
gpkit.small_scripts.nomial_latex_helper(c, pos_vars, neg_vars)
     Combines (varlatex, exponent) tuples, separated by positive vs negative exponent, into a single latex
     string
```

11.1.10 gpkit.solution_array module

gpkit.small_scripts.veckeyed(key)
 Return a veckey version of a VarKey

gpkit.small_scripts.try_str_without(item, excluded)
Try to call item.str_without(excluded); fall back to str(item)

Defines SolutionArray class

```
class gpkit.solution_array.SolutionArray
     Bases: gpkit.small_classes.DictOfLists
     A dictionary (of dictionaries) of lists, with convenience methods.
     cost: array variables: dict of arrays sensitivities: dict containing:
          monomials: array posynomials: array variables: dict of arrays
     localmodels [NomialArray] Local power-law fits (small sensitivities are cut off)
     >>> import gpkit
     >>> import numpy as np
     >>> x = gpkit.Variable("x")
     >>> x_min = gpkit.Variable("x_{min}", 2)
     >>> sol = gpkit.Model(x, [x >= x_min]).solve(verbosity=0)
     >>>
     >>> # VALUES
     >>> values = [sol(x), sol.subinto(x), sol["variables"]["x"]]
     >>> assert all(np.array(values) == 2)
     >>> # SENSITIVITIES
     >>> senss = [sol.sens(x_min), sol.sens(x_min)]
     >>> senss.append(sol["sensitivities"]["variables"]["x_{min}"])
     >>> assert all(np.array(senss) == 1)
     plot (posys=None, axes=None)
          Plots a sweep for each posy
     program = None
     subinto (posy)
          Returns NomialArray of each solution substituted into posy.
     summary (showvars=(), ntopsenss=5)
          Print summary table, showing top sensitivities and no constants
     table (showvars=(), tables=('cost', 'sweepvariables', 'freevariables', 'constants', 'sensi-
             tivities'), **kwargs)
          A table representation of this SolutionArray
          tables: Iterable
              Which to print of ("cost", "sweepvariables", "freevariables", "constants", "sensitiv-
          fixedcols: If true, print vectors in fixed-width format latex: int
              If > 0, return latex format (options 1-3); otherwise plain text
          included_models: Iterable of strings If specified, the models (by name) to include
          excluded_models: Iterable of strings If specified, model names to exclude
          str
     table_titles = {'constants': 'Constants', 'freevariables': 'Free Variables', 'sw
gpkit.solution_array.insenss_table(data,_, maxval=0.1, **kwargs)
     Returns insensitivity table lines
```

```
qpkit.solution_array.results_table (data, title, minval=0, printunits=True, fixed-
                                                 cols=True, varfmt='%s: ', valfmt='%-.4g
                                                 ', vecfmt='%-8.3g', included models=None,
                                                 excluded_models=None, latex=False, sortby-
                                                 vals=False, ** )
     Pretty string representation of a dict of VarKeys Iterable values are handled specially (partial print-
     ing)
     data: dict whose keys are VarKey's data to represent in table
     title: string minval: float
          skip values with all(abs(value)) < minval
     printunits: bool fixedcols: bool
          if True, print rhs (val, units, label) in fixed-width cols
     varfmt: string format for variable names
     valfmt: string format for scalar values
     vecfmt: string format for vector values
     latex: int If > 0, return latex format (options 1-3); otherwise plain text
     included_models: Iterable of strings If specified, the models (by name) to include
     excluded_models: Iterable of strings If specified, model names to exclude
     sortbyvals [boolean] If true, rows are sorted by their average value instead of by name.
gpkit.solution_array.senss_table(data,
                                                        showvars=(),
                                                                         title='Sensitivities',
                                              **kwargs)
     Returns sensitivity table lines
gpkit.solution_array.topsenss_filter(data, showvars, nvars=5)
     Filters sensitivities down to top N vars
gpkit.solution_array.topsenss_table(data, showvars, nvars=5, **kwargs)
     Returns top sensitivity table lines
11.1.11 gpkit.varkey module
Defines the VarKey class
class gpkit.varkey.VarKey(name=None, **kwargs)
     Bases: object
     An object to correspond to each 'variable name'.
     name [str, VarKey, or Monomial] Name of this Variable, or object to derive this Variable from.
     **kwargs: Any additional attributes, which become the descr attribute (a dict).
     VarKey with the given name and descr.
     latex (excluded=None)
          Returns latex representation.
     latex_unitstr()
          Returns latex unitstr
```

```
naming
    Returns this varkey's naming tuple

classmethod new_unnamed_id()
    Increment self.count and return it

str_without (excluded=None)
    Returns string without certain fields (such as 'models').

subscripts = ('models', 'idx')

unitstr(into='%s', options='~', dimless=")
    Returns the string corresponding to an object's units.
```

11.1.12 Module contents

GP and SP modeling package

CHAPTER 12

Citing GPkit

If you use GPkit, please cite it with the following bibtex:

```
@Misc{gpkit,
    author={Edward Burnell},
    title={GPkit software for geometric programming},
    howpublished={\url{https://github.com/convexengineering/gpkit}},
    year={2018},
    note={Version 0.7.0}
}
```

CHAPTER 13

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- Marshall Galbraith for setting up continuous integration.
- Stephen Boyd for inspiration and suggestions.
- Kirsten Bray for designing the GPkit logo.

CHAPTER 14

Release Notes

This page lists the changes made in each point version of gpkit.

14.1 Version 0.7.0

- Variable's values are now used only in their first ConstraintSet; in other ConstraintSets they're free variables
- Variable values can be preserved by setting constant=True during variable declaration
- MOSEK home directory can be set by a MSKHOME environment variable at build time
- sol (var) now always returns Pint Quantities, even if the variable is dimensionless
- ullet sol[...] [var], on the other hand, now always returns floats / numpy arrays of floats
- Optional boundedness checking in docstring (see usage in docs)
- Automatic boundedness checking for GPs
- Sankey diagrams
- · Many other fixes

14.2 Version 0.6.0

- new much faster NomialMap data structure (#682)
- Many bug fixes and small improvements.
- 6900 lines of code, 2200 lines of tests, 2100 lines of docstring.

14.3 Version 0.5.3

- faster SP solves (#1109)
- LinkedConstraintSet deprecated (#1110)
- Fixes to autosweep, ConstraintSet, interactive
- Solution time is now stored with soltutions (including sweeps/SPs)
- Model strings are divided with slashes (e.g. Airplane/Wing)

14.4 Version 0.5.2

- Added new sweep and autosweep methods to Model
 - Added plot routines to the results of those routines to make it easy to plot a 1D sweep.
- Added new summary method to solution_array.
 - It and table accept iterables of vars, will only print vars in that iterable (or, by default, all vars)
- Cleaned up and documented the interactive submodule
 - removed contour and sensitivity plots
 - added a 1D-sweep plotting function
 - added that plotting function as an option within the control panel interface
- Overhauled and documented three types of variables whose value is determined by functions:
 - calculated constants
 - post-solve calculated variables
 - between-GP-solves calculated variables (for Sequential Geometric Programs)
- Fix Bounded and implement debug () for SPs
- Apply subinplace to substitutions dictionary as well
- Require GP substitutions to be Numbers only
- Extend Bounded to one-sided bounds
- Print model's numbers by default, unless "modelnums" in exclude
- Implement lazy keymapping, allowing GP/SP results to be KeyDicts
- Handle Signomial Inequalities that become Posynomial Inequalities after substitution
- · Various documentation updates
- Various bug fixes

14.5 Version 0.5.1

• O(N) sums and monomial products

- Warn about invalid ConstraintSet elements
- allow setting Tight tolerance as a class attribute
- full backwards compatibility for __init__ methods
- scripts to test remote repositories
- minor fixes, tests, and refactors
- 3550 lines of code, 1800 lines of tests, 1700 lines of docstring. (not counting interactive)

14.6 Version 0.5.0

- No longer recommend the use of linked variables and subinplace (see below)
- · Switched default solver to MOSEK
- Added Linked Variable diagram (PR #915)
- Changed how overloaded operators interact with pint (PR #938)
- Added and documented debugging tools (PR #933)
- · Added and documented vectorization tools
- Documented modular model construction
- 3200 lines of code, 1800 lines of tests, 1700 lines of docstring. (not counting interactive)

14.6.1 Changes to named models / Model inheritance

We are deprecating the creation of named submodels with custom __init__ methods. Previously, variables created during __init__ in any class inheriting from Model were replaced by a copy with __class__.__name__ added as varkey metadata. This was slow, a bit irregular, and hacky.

We're moving to an explicitly-irregular setup method, which (if declared for a class inheriting from Model) is automatically called during Model. __init__ inside a NamedVariables (self. __class__._name__) environment. This 1) handles the naming of variables more explicitly and efficiently, and 2) allows us to capture variables created within setup, so that constants that are not a part of any constraint can be used directly (several examples of such template models are in the new *Building Complex Models* documentation).

Model.__init__ calls setup with the arguments given to the constructor, with the exception of the reserved keyword substitutions. This allows for the easy creation of a named model with custom parameter values (as in the documentation's Beam example). setup methods should return an iterable (list, tuple, ConstraintSet, ...) of constraints or nothing if the model contains no constraints. To declare a submodel cost, set self.cost during setup. However, we often find declaring a model's cost explicitly just before solving to be a more legible practice.

In addition to permitting us to name variables at creation, and include unconstrained variables in a model, we hope that setup methods will clarify the side effects of named model creation.

14.7 Version 0.4.2

• prototype handling of SignomialEquality constraints

14.6. Version 0.5.0 97

- fix an issue where solution tables printed incorrect units (despite the units being correct in the SolutionArray data structure)
- fix controlpanel slider display for newer versions of ipywidgets
- fix an issue where identical unit-ed variables could have different hashes
- Make the text of several error messages more informative
- Allow monomial approximation of monomials
- bug fixes and improvements to TightConstraintSet
- Don't print results table automatically (it was unwieldy for large models). To print it, print sol.table().
- Use cvxopt's ldl kkt solver by default for more robustness to rank issues
- Improved ConstraintSet. __getitem__, only returns top-level Variable
- Move toward the varkeys of a ConstraintSet being an immutable set
- CPI update
- numerous pylint fixes
- · BoundedConstraint sets added for dual feasibility debugging
- SP sweep compatibility

14.8 Version 0.4.0

- New model for considering constraints: all constraints are considered as sets of constraints which may contain other constraints, and are asked for their substitutions / posynomial less than 1 representation as late as possible.
- Support for calling external code during an SP solve.
- New class KeyDict to allow referring to variables by name or with objects.
- Many many other bug fixes, speed ups, and refactors under the hood.

14.9 Version 0.3.4

- Modular / model composition fixes and improvements
- Working controlpanel() for Model
- · ipynb and numpy dependency fixes
- printing fixes
- · El Capitan fix
- slider widgets now have units

14.10 Version 0.3.2

- · Assorted bug fixes
- Assorted internal improvements and simplifications
- · Refactor signomial constraints, resulting in smarter SP heuristic
- Simplify and strengthen equality testing for nomials
- Not counting submodules, went from 2400 to 2500 lines of code and from 1050 to 1170 lines of docstrings and comments.

14.11 Version 0.3

- Integrated GP and SP creation under the Model class
- Improved and simplified under-the-hood internals of GPs and SPs
- New experimental SP heuristic
- Improved test coverage
- Handles vectors which are partially constants, partially free
- Simplified interaction with Model objects and made it more pythonic
- Added SP "step" method to allow single-stepping through an SP
- Isolated and corrected some solver-specific behavior
- Fully allowed substitutions of variables for 0 (commit 4631255)
- Use "with" to create a signomials environment (commit cd8d581)
- Continuous integration improvements, thanks @galbramc!
- Not counting subpackages, went from 2200 to 2400 lines of code (additions were mostly longer error messages) and from 650 to 1050 lines of docstrings and comments.
- · Add automatic feasibility-analysis methods to Model and GP
- Simplified solver logging and printing, making it easier to access solver output.

14.12 Version 0.2

- · Various bug fixes
- Python 3 compatibility
- Added signomial programming support (alpha quality, may be wrong)
- Added composite objectives
- Parallelized sweeping
- Better table printing
- Linked sweep variables
- · Better error messages

14.10. Version 0.3.2

- Closest feasible point capability
- Improved install process (no longer requires ctypesgen; auto-detects MOSEK version)
- Added examples: wind turbine, modular GP, examples from 1967 book, maintenance (part replacement)
- Documentation grew by ~70%
- Added Advanced Commands section to documentation
- Many additional unit tests (more than doubled testing lines of code)

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